Axion decay constants at special points in type II superstring theory

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Axion

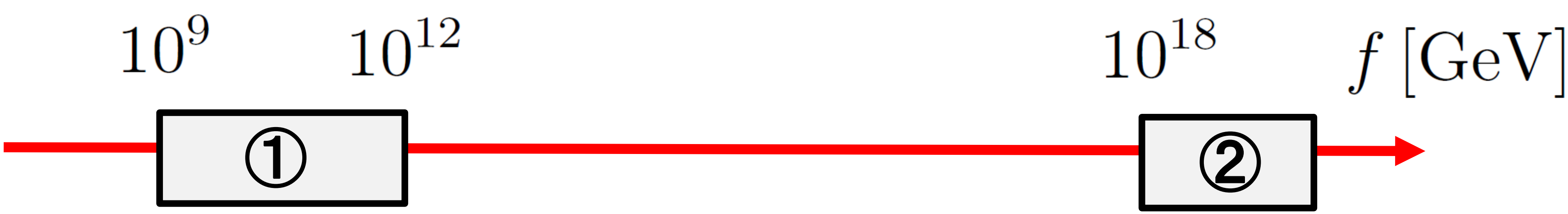
$$\mathcal{L} = \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{a}{16\pi^2 f}\mathrm{tr}(F^{\mu\nu}\tilde{F}_{\mu\nu})$$

○ Solution of the strong CP problem

R. D. Peccei and Helen R. Quinn ('77)

$$|\theta| < 10^{-10}$$

① The candidate of dark matter ("QCD axion window")
② Successful axion inflation (natural inflation etc.)



Axions in string theory

A lot of axions originating from high-dimensional form fields:

e.g.

○ Integrating the 2-form A over the internal 2-cycle Σ of internal manifold,

$$a^i(x) = \int_{\Sigma^i} A$$

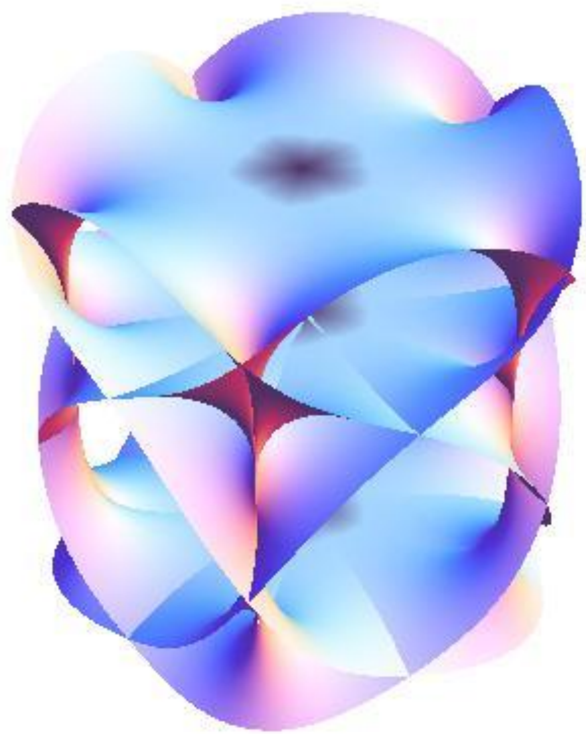
Axions 2-form

Perturbative flat potential (shift symmetry)

○Gauge symmetry: $A \to A + d\Lambda$ becomes a shift symmetry for $a^i(x)$, $a^i(x) \to a^i(x)$ +const.

Closed string axions in superstring on Calabi-Yau (CY)

In the low-energy effective theory,
closed string axions are accompanied by the closed string moduli.



Closed string moduli (Scalar fields)

i) Dilaton (S)

ii) Kähler moduli (T)
: Size of the internal cycles

iii) Complex structure moduli (z)
: Shape of CY

Typical axion decay constant in string theory

Classical formula for Kähler axion:

$$f \simeq \frac{M_{\rm Pl}}{\mathcal{V}^n}$$

$\mathcal{V}$: Calabi-Yau volume
$M_{\rm Pl}$: reduced Planck mass

$n = 2/3$:Type II string
$n = 1/6$:Heterotic string

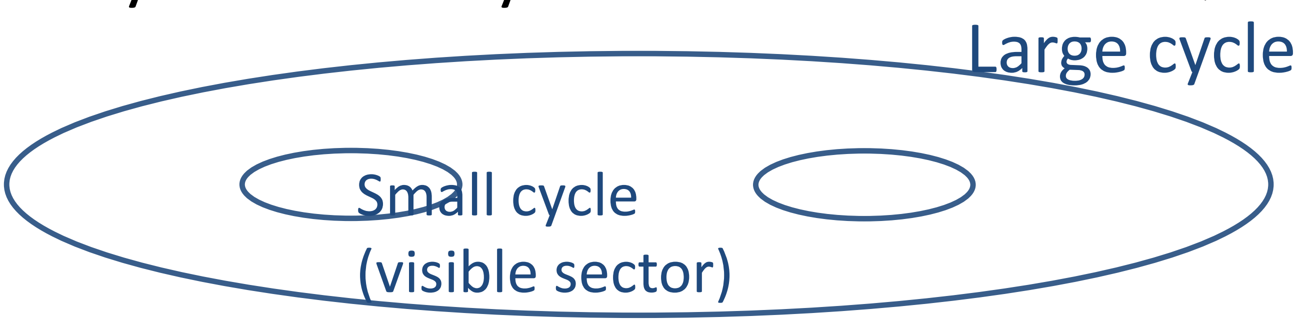
Observed gauge couplings
➔ Constrained axion decay constant

$$10^{16} \leq f \leq 10^{17}[\mathrm{GeV}]$$

K. Choi and J. E. Kim, ('85);
T. Banks, M. Dine, P. J. Fox, and E. Gorbatov('03);
P. Svrcek and E. Witten, ('06).

Large (mild) volume of CY
➔ Tiny axion decay constant

V. Balasubramanian, P. Berglund, J. P. Conlon, and F. Quevedo ('05)



Axion decay constant in string theory

$$f \simeq \frac{M_{\rm Pl}}{\mathcal{V}^n}$$

$\mathcal{V}$: Calabi-Yau volume
$M_{\rm Pl}$: reduced Planck mass

Large (mild) volume of CY
➔ Suppressed loop-and non-perturbative corrections

When the volume of CY is of $O(1)$,
the loop-and non-perturbative corrections are important.

$$K = -2M_{\rm Pl}^2 \ln\left(\mathcal{V} + \Delta\mathcal{V}\right)$$

For type IIA string on CY orientifold,
even for $O(1)$ volume of CY,
instanton corrections➔ Planckian decay constant

$$f = \sqrt{2K_{T\bar{T}}}$$

J. P. Conlon and S. Krippendorf, ('16).

$f \simeq 3M_{\rm Pl}$ for specific CY manifold

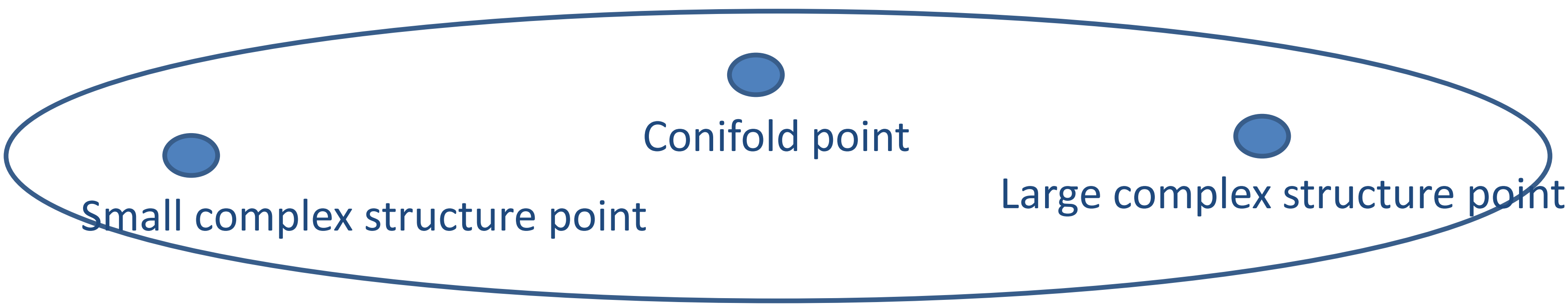
The axions associated with the complex structure moduli ?

Quantum corrections for the complex structure moduli ?

The axions associated with the complex structure moduli ?

We proceed to study the detail of the quantum and geometrical corrections for the axion decay constant.

In particular, we focus on the type IIB string on CY orientifold. Such corrections are exactly calculated by the topological string.



The decay constant will be irrelevant to the string scale.

$$M_s \simeq \frac{g_s}{\sqrt{4\pi\mathcal{V}}} M_{\rm Pl}$$

Outline

- Axion
- Typical axion decay constant
- Complex structure axion
- Conclusion

Axions associated with the complex structure (CS) moduli

The Kähler potential for CS moduli (z) in type IIB string on CY:

$$K = -\ln\left[i\Pi^\dagger \cdot \Sigma \cdot \Pi\right]$$

Σ : Symplectic matrix

Period vector over the three-cycles on CY: (A_a, B^a)

$$\Pi^t(z) = \left(\int_{A_a}\Omega, \int_{B^a}\Omega\right)$$

Ω : holomorphic three-form

Geometrical sym. of Ω induces the existence of axions.

Axions associated with the complex structure (CS) moduli

Under the monodromy trf. around the special point z_{sp},

$$\Pi_i(ze^{2\pi i}) = T_{ij}[z_{\mathrm{sp}}]\Pi_j(z)$$

Kähler potential is invariant. T : Monodromy matrix

Such symmetries of the Kähler potential
➔axionic shift symmetries

<u>Axions naturally appear in the low-energy effective theory.</u>

We focus on
the 14 CY manifolds with single complex structure modulus.

Y. -H. Chen, Y. Yang and N. Yui, J. Reine Angew ('08)

Period vector is obtained by solving the Picard-Fuchs(PF) eq.

$$\left\{\delta^4 - z(\delta+\alpha_1)(\delta+\alpha_2)(\delta+\alpha_3)(\delta+\alpha_4)\right\}\Pi_i(z) = 0$$

$$(i = 1, 2, 3, 4)$$

$$\delta = zd/dz$$

α_i are the rational numbers depending on the CY data.

E.g., Mirror Quintic CY manifold, $CP_{1,1,1,1,1}[5]$

$$\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \frac{1}{5}$$

S. Hosono, A. Klemm, S. Theisen, S.-T. Yau ('94)

Picard-Fuchs(PF) eq.:

$$\left\{\delta^4 - z(\delta+\alpha_1)(\delta+\alpha_2)(\delta+\alpha_3)(\delta+\alpha_4)\right\}\Pi_i(z) = 0$$

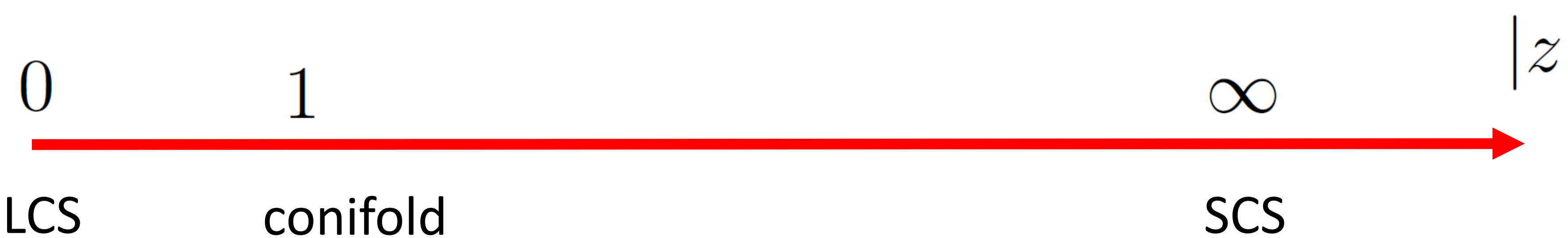
"Special points": regular singular points of PF eq.

We focus on the several special points of moduli space.

i) Small complex structure (SCS) point ($z \sim \infty$)

ii) Conifold point ($z \sim 1$)

iii) Large complex structure (LCS) point ($z \sim 0$)



Kähler potential around the SCS point($z \sim \infty$)

In all cases, axion (arg(z)) appears around the SCS point.

$$\alpha_1 \neq \alpha_2 \neq \alpha_3 \neq \alpha_4$$

$$K_{\mathrm{SCS}} = -\ln\left[\sum_{k=1}^{4} A_k |z|^{-2\alpha_k}\right] \qquad T^d = \mathbf{1}$$

d: LCM of the degree of polynomials

$$\alpha_1 \neq \alpha_2 \neq \alpha_3 = \alpha_4$$

$$K_{\mathrm{SCS}} = -\ln\left[\sum_{k=1}^{2} A_k |z|^{-2\alpha_k} - B_3 |z|^{-2\alpha_3} \ln|z|\right] \qquad (T^d - 1)^2 = 0$$

$$\alpha_1 = \alpha_2 \neq \alpha_3 = \alpha_4$$

$$K_{\mathrm{SCS}} = -\ln\left[C_1 |z|^{-2\alpha_1} \ln|z| + C_3 |z|^{-2\alpha_3} \ln|z|\right] (T^d - 1)^2 = 0$$

$$\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4$$

$$K_{\mathrm{SCS}} = -\ln\left[D_1 |z|^{-2\alpha_1} (\ln|z|)^3\right] \qquad (T - \mathbf{1})^4 = 0$$

Axion decay constant around the SCS point

In all cases,
massless axion (arg(z)) appears around the SCS point.
There is a continuous shift symmetry, arg(z)→arg(z) +const.

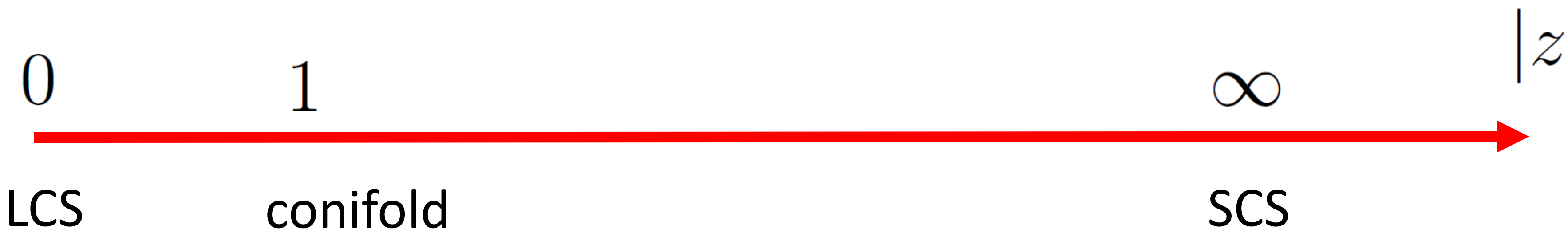
The axion Kähler metric vanishes at the SCS point($z \sim \infty$).

$$K_{\theta\theta} = K_{z\bar{z}}|z|^2 \to 0 \qquad \theta = \arg(z)$$

<u>Tiny axion decay constant is realized around the SCS point.</u>

Axion decay constant around special points

In a similar fashion, we find that
the axion decay constant vanishes at all the special points.



When the CS modulus is stabilized around the special points,
we can realize the tiny axion decay constant.

Massless axion and moduli stabilization

Procedures to stabilize the CS modulus around the SCS point:
We redefine the CS modulus as $z^{-\alpha_1} \to z$.

Kähler potential:

$$K = -\ln(S+\bar{S}) + K(T+\bar{T}) + K(|z|^2)$$

Non-perturbatively generated superpotential:

$$W = W(S,T)$$

S : Dilaton
T : Kähler moduli

F-term scalar potential:

$$V_F = e^K (K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3|W|^2)$$

Uplifting potential induced by the anti D-brane:

$$V_{\rm up} = e^{2K/3} \mathcal{P}(T+\bar{T}, |z|^2)$$

Massless axion and moduli stabilization

Total scalar potential:

$$V = V_F + V_{\rm up}$$

By assuming the following condition (as in the KKLT scenario),

$$\langle V_{\rm up} \rangle \simeq 3m_{3/2}^2$$

The extremal condition of CS leads to the VEV of CS,

$$|z|^{4\frac{\alpha_2}{\alpha_1}-5} \sim \partial_z (\ln \mathcal{P})$$

The massless is then obtained. $\theta = \arg(z)$

Other moduli fields are stabilized by the superpotential.

Massless axion and moduli stabilization

E.g., for CY manifold defined in $CP^5_{3,2,2,1,1,1}[4,6]$

$f \simeq 10^{12}\,\mathrm{GeV}$ is achieved under $\partial_z(\ln \mathcal{P}) \simeq 3 \times 10^{-7}$

Thus, decay constant of massless axion is much smaller than the string scale.

So far, we have assumed the tiny value of $\partial_z(\ln \mathcal{P})$.

It is interesting and challenging issue to identify the specific form of $\mathcal{P}$, since it depends on the detailed D-brane setup.

Conclusion

We study the decay constant of closed string axion around the special points of moduli space in type II string on CY manifolds.

Quantum or geometrical corrections are important for the decay constants of closed string axions.

Tiny axion decay constant can be realized around all the special points (SCS, conifold and LCS points).

We also propose the moduli stabilization leading to the massless axion.

Appendix

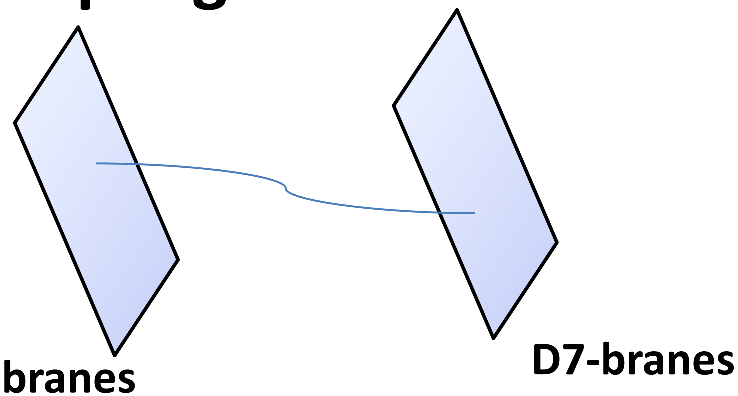
Comments on the axionic coupling

The axion of complex structure moduli appears in the gauge coupling at the one-loop level.

○Type IIB string on toroidal orientifold and orbifold with D3/D7-branes

$T^6/(Z_2 \times Z_2)$

The gauge coupling on D7-branes:



D. Lüst and S. Stieberger, ('03)

$$\frac{1}{g^2} = \frac{T}{4\pi} + \frac{\Delta(U)}{16\pi^2}$$

U **:Complex structure moduli (Shape of torus)**

T **:Kähler moduli (Volume of torus)**

Moduli-dependent threshold corrections

PF eq.

$$\left\{\delta^4 - z(\delta+\alpha_1)(\delta+\alpha_2)(\delta+\alpha_3)(\delta+\alpha_4)\right\}\Pi_i(z) = 0$$

$$(i = 1, 2, 3, 4)$$

14 CY manifolds defined in weighted projective space.

Ambient spaces	$(\alpha_1, \alpha_2, \alpha_3, \alpha_4)$	Degeneracies of α_i
$CP^4_{1,1,1,1,1}[5]$	$(1/5, 2/5, 3/5, 4/5)$	0
$CP^4_{2,1,1,1,1}[6]$	$(1/6, 2/6, 4/6, 5/6)$	0
$CP^4_{4,1,1,1,1}[8]$	$(1/8, 3/8, 5/8, 7/8)$	0
$CP^4_{5,2,1,1,1}[10]$	$(1/10, 3/10, 7/10, 9/10)$	0
$CP^5_{6,4,1,1,1,1}[2,12]$	$(1/12, 5/12, 7/12, 11/12)$	0
$CP^5_{3,2,2,1,1,1}[4,6]$	$(1/4, 1/3, 2/3, 3/4)$	0
$CP^5_{2,1,1,1,1,1}[3,4]$	$(1/6, 1/4, 3/4, 5/6)$	0

PF eq.

$$\left\{\delta^4 - z(\delta+\alpha_1)(\delta+\alpha_2)(\delta+\alpha_3)(\delta+\alpha_4)\right\}\Pi_i(z) = 0$$

$$(i = 1, 2, 3, 4)$$

14 CY manifolds defined in weighted projective space.

Ambient spaces	$(\alpha_1, \alpha_2, \alpha_3, \alpha_4)$	Degeneracies of α_i
$CP^5_{1,1,1,1,1,1}[2,4]$	$(1/4, 3/4, 1/2, 1/2)$	1
$CP^6_{1,1,1,1,1,1,1}[2,2,3]$	$(1/3, 2/3, 1/2, 1/2)$	1
$CP^5_{3,1,1,1,1,1}[2,6]$	$(1/6, 5/6, 1/2, 1/2)$	1
$CP^5_{1,1,1,1,1,1}[3,3]$	$(1/3, 1/3, 2/3, 2/3)$	2
$CP^5_{2,2,1,1,1,1}[4,4]$	$(1/4, 1/4, 3/4, 3/4)$	2
$CP^5_{3,3,2,2,1,1}[6,6]$	$(1/6, 1/6, 5/6, 5/6)$	2
$CP^7_{1,1,1,1,1,1,1,1}[2,2,2,2]$	$(1/2, 1/2, 1/2, 1/2)$	4

Meijer G function:

$$U_j(z) = \frac{1}{(2\pi i)^{j+1}} \int_C ds \frac{\Gamma(-s)^{j+1} \Pi_{i=1}^4 \Gamma(s+\alpha_i)}{\Gamma(s+1)^{3-j} \Pi_{i=1}^4 \Gamma(\alpha_i)} \left((-1)^{j+1} z\right)^s$$

with $j = 0, 1, 2, 3$. In the SCS point, the contour C is taken to extend from $-i\infty$ to $i\infty$ so as to enclose the poles $s = -\alpha_i - n$ with n being non-negative integer.