Chiral response in a p-wave superconductor

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Contents

- Chiral p-wave superconductor

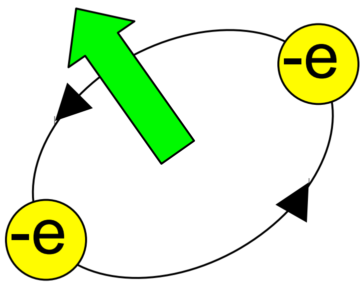
  breaking T and *P*

- Induced Chern-Simons-like term
- Physical implications

Chiral *p*-wave superconductor

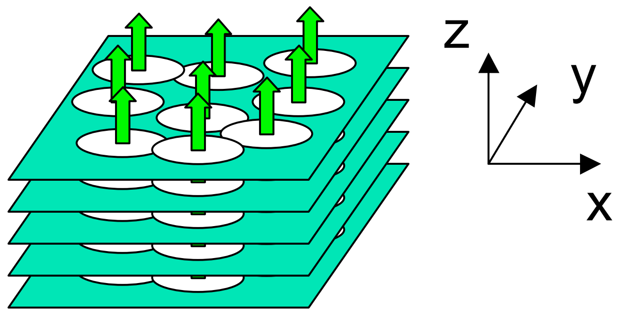
- Cooper pair
  → superconductivity

- *relative angular momentum* $\boldsymbol{L}_{rel}$



- <u>*Chiral p-wave state*</u>
  $\boldsymbol{L}_{rel}$ of pairs are alined
  "*ferromagnetically*"

In a layered system…
$L_{rel\ z}$=1 (or -1)

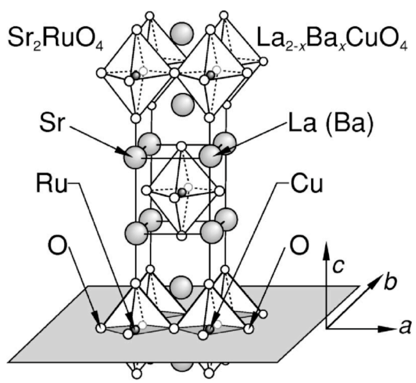


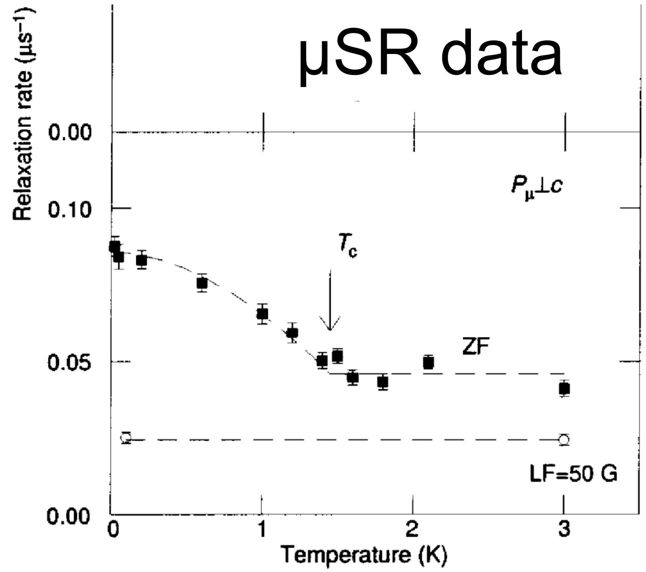
Sr_2RuO_4

- Chiral p-wave;

  the most plausible pairing state in a layered superconductor Sr_2RuO_4

  Cf) Mackenzie-Maeno, RMP ('03)



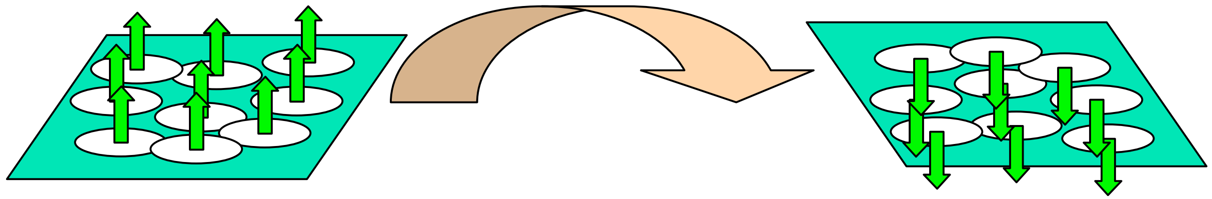


Spontaneous magnetization as an evidence for time-reversal symmetry breaking

The state breaks *T* and P_{2+1D}

$$T;\ t \rightarrow -t, \quad P_{2+1D};\ x \rightarrow -x,\ y \rightarrow y$$

$$\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{p} \ \rightarrow\ -\boldsymbol{L}$$



T and P_{2+1D} breaking …

- From the point of view of symmetry breaking, the chiral *p*-wave state resembles the massive Dirac QED in 2+1D

- The Chern-Simons term

universal

$$\frac{1}{2}\frac{e^2}{4\pi}\varepsilon_{\mu\rho\nu}A_\mu\partial_\rho A_\nu$$

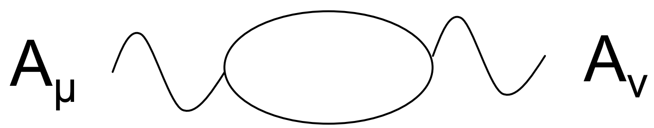
is induced in the low energy effective action for the U(1) gauge field

Cf) Daser-Jackiw-Templeton, K. Ishikawa, Niemi & Semenoff, Redlich…etc

- In the chiral *p*-wave superconductor ???

Induced Chern-Simons-like term in a chiral p-wave SC

- We evaluate 1-loop diagram of the Bogoliubov quasiparticle in the chiral *p*-wave state and obtain the low energy effective action



1-loop effective action includes a Chern-Simons-like term

$$\frac{a}{2}\varepsilon_{ij}(A_0\partial_i A_j + A_i\partial_j A_0)$$

Ref) G.E.Volovik ('88),
JG＆K.Ishikawa ('98)('99)

Compared to the Chern-Simons term $\frac{1}{2}\frac{e^2}{4\pi}\varepsilon_{\mu\rho\nu}A_\mu\partial_\rho A_\nu$ in massive Dirac QED_{2+1},

- $\varepsilon_{ij}A_i\partial_0 A_j$ is absent.

- The total action is gauge invariant due to the Nambu-Goldstone mode *θ* appeared in the form *A+dθ*

- The coefficient $a = \frac{e^2}{4\pi}$ (particle-hole symmetric case)

Corrections from impurity scattering

- Colleman-Hill's non-renormalization theorem* for the C-S term in Dirac QED_{2+1} is ***not applied*** to the chiral p-wave case because of the U(1) gauge symmetry breaking

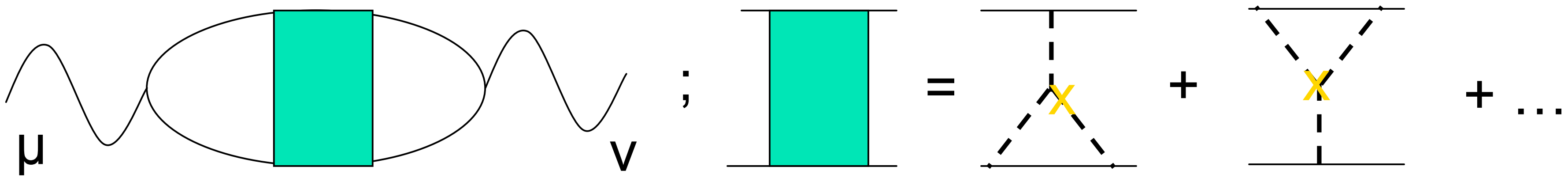
* For IQHE system;
Imai-Ishikawa-Matsuyama-Tanaka
PRB ('90)

- Then, higher-order corrections would play an important role

$$\frac{b}{2}\varepsilon_{ij}A_i\partial_0 A_j$$

is induced by vertex corrections from *impurities*

JG, Phys. Rev. B (2008)



2nd-Born type diagrams

• the coefficient b is non-universal

• the coefficient a receives a non-universal correction

$$a = \frac{e^2}{4\pi} + \dots$$

Integrating out NG mode θ

→JG &K. Ishikawa, Phys Lett A ('98)('99)
JG, PRB('08),LT25-Proceeding ('08)

- Effective Lagrangian

$$L_{eff} = \frac{C}{2{v_F}^2} A_0^{T\,2} - \frac{C}{2} A_i^{T\,2} + \frac{a}{2}\varepsilon_{ij}(A_0^T \partial_i A_j^T + A_i^T \partial_j A_0^T) + \frac{b}{2}\varepsilon_{ij} A_i^T \partial_0 A_j^T$$

"transverse" components

$$\begin{cases} A_0^T = A_0 - \dfrac{\partial_0}{\partial_0^2 - v_F^2 \boldsymbol{\partial}^2}(\partial_0 A_0 - v_F^2 \boldsymbol{\partial}\cdot\boldsymbol{A}) \\ A_i^T = A_i - \dfrac{\partial_i}{\partial_0^2 - v_F^2 \boldsymbol{\partial}^2}(\partial_0 A_0 - v_F^2 \boldsymbol{\partial}\cdot\boldsymbol{A}) \end{cases}$$

v_F; Fermi velocity divided by the light velocity

This is the same form as the effective Lagrangian of *anyon superconductivity*

Chen-Witten-Wilczek-Halperin ('89), N. Nagaosa (private comm.)

Gauge-invariant linear response of chiral p-wave SC

JG & K.Ishikawa,Phys. Lett.A ('98)('99)
Luchyn-Nagornykh-Yakovenko,PRB('08)
Roy & Kallin,PRB('08),
JG, PRB ('08);LT25-Proceedings('08)

$$j_x(p) = \frac{-C}{ip_0}\left\{\frac{p_0^{\;2}}{p_0^{\;2} - v_F^{\;2}\mathbf{p}^2}\right\}E_x(p) + \left\{a\frac{-v_F^{\;2}\mathbf{p}^2}{p_0^{\;2} - v_F^{\;2}\mathbf{p}^2} + b\frac{p_0^{\;2}}{p_0^{\;2} - v_F^{\;2}\mathbf{p}^2}\right\}E_y(p)$$

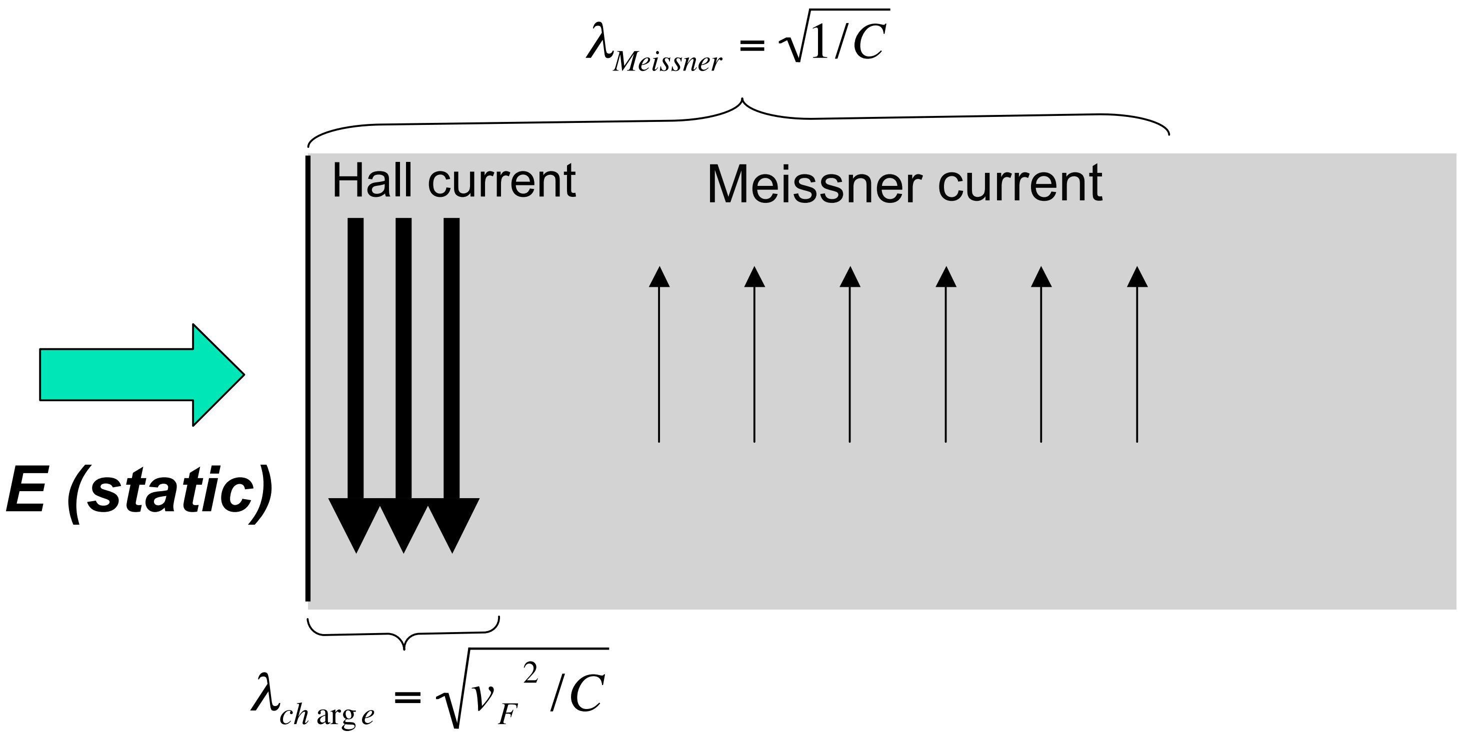
σ_{xx} usual behavior of SC

σ_{xy} chiral response peculiar to the P- and T-violating SC

(i) $\mathbf{E} = \mathbf{E}(p_0 = 0, \mathbf{p} \neq 0) \longrightarrow \sigma_{xy} = \underline{a}$

(ii) $\mathbf{E} = \mathbf{E}(p_0 \neq 0, \mathbf{p} = 0) \longrightarrow \sigma_{xy} = \underline{b}$

Zero-field Hall effect ?



$\lambda_{charge} << \lambda_{Meissner}$ ∴Signal appears to be suppressed

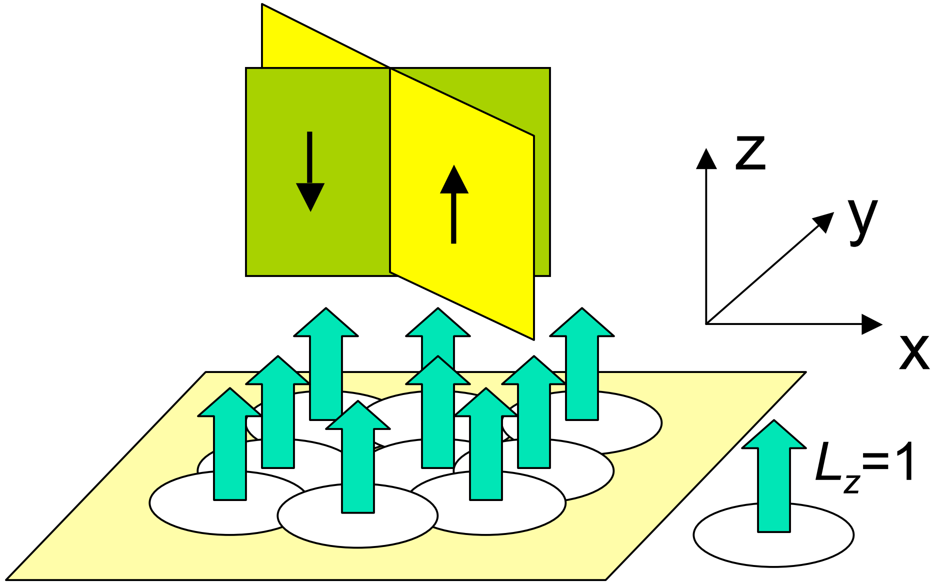
Cf) Furusaki-Matsumoto-Sigrist ('01)

Kerr rotation … well-known as an effect of ferromagnet

• incident light … (i) linealy polarized, (ii) **p** // z

• the polarization of reflected light is rotated

$\frac{b}{2}\varepsilon_{ij}A_i\,\partial_0 A_j$ -term yields the rotation since this term mixes A_x and A_y

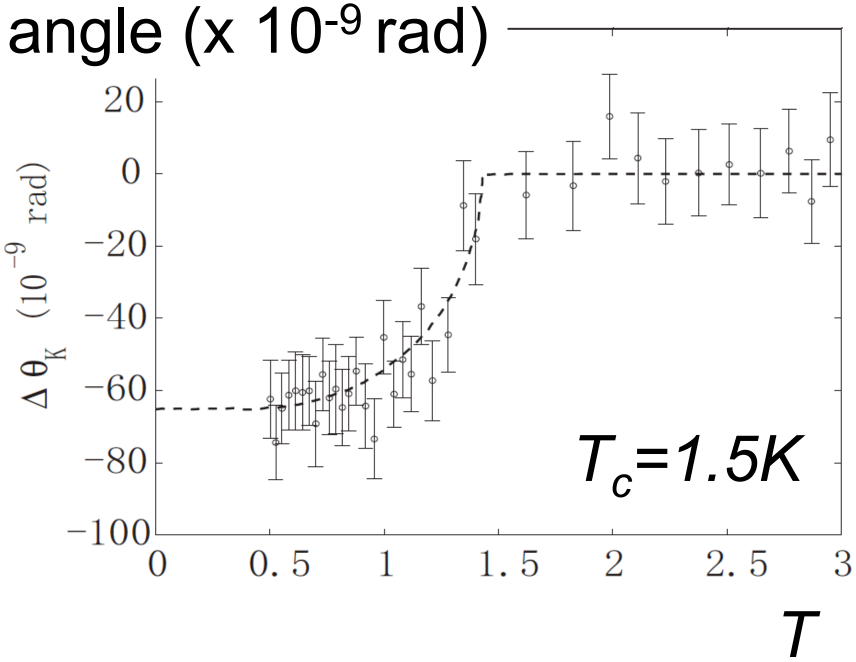


Cf) QHE; K. Ishikawa, PRB('85), anyonSC; Wen-Zee PRL('88),PRB('89)

Measurement of Kerr rotation in Sr_2RuO_4

Xia et al PRL ('06)

Kerr rotation angle (x 10^{-9} rad)



- The laser-frequency

ω=0.8eV >> $|\Delta|=10^{-4}$ eV

∴ we should go beyond the low energy approach, i.e., $b \rightarrow b(p_0)$

JG, PRB ('08), LT25-Proceedings ('08)

- estimated value is *about* *10^{-3} smaller* than the observed value

Summary

- We consider electromagnetic response of a chiral (*T*- and *P*-breaking) p-wave superconductor

- The Chern-Simons-like term is induced in the effective Lagrangian, which has the same form with the effective Lagrangian of the anyon superconductor.

- The C-S-like term gives chiral responses
  e.g., zero-field Hall effect, Kerr rotation

II. zero-field Hall conductivity

- Quasiparticle Hamiltonian;

$$\tilde{\mathscr{H}}[A'] = \tilde{\Psi}^{\dagger} \begin{pmatrix} \varepsilon(\boldsymbol{p}+eA') & \Delta(\boldsymbol{p}) \\ \Delta^{\dagger}(\boldsymbol{p}) & -\varepsilon(\boldsymbol{p}-eA') \end{pmatrix} \tilde{\Psi},$$

$$A'_{\mu} = A_{\mu} + \frac{1}{e}\partial_{\mu}\theta. \qquad (8)$$

$\Delta(\boldsymbol{p}) \propto p_x + i p_y$ (chiral p-wave; $L_z=1$, TRS breaking state)

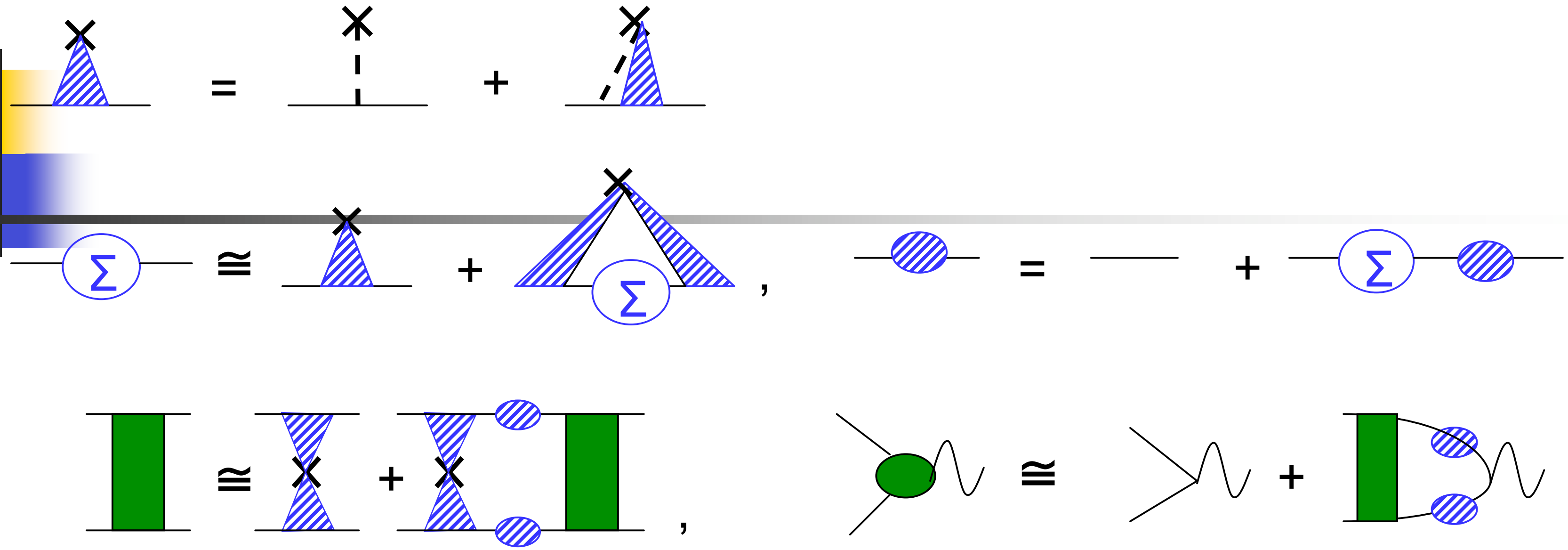
A_μ・・・gauge field

Ψ・・・Fermion,

θ・・・goldstone mode (phase degree of freedom of $\Delta(\boldsymbol{p})$)

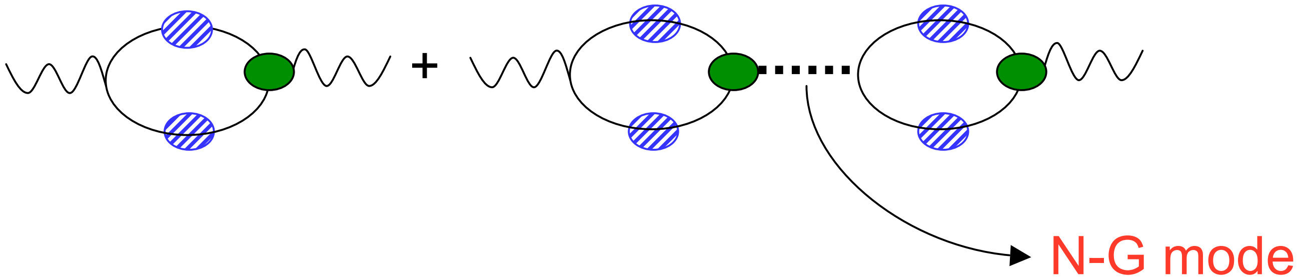
We also include <u>impurities (*point-like*)</u>

impurity corrections



gauge-invariant formalism for current-current correlation (Y. Nambu ('60))

$\pi_{ij}(\omega, \mathbf{q})$ =



Kubo formula

$$\sigma_{xy}(\omega, \mathbf{q}) = \frac{1}{i\omega}\left[\frac{1}{2!}\epsilon_{ij}\pi_{ij}(\omega, \mathbf{q})\right]$$

Zero-field Hall conductivity in chiral p-wave state

$$\sigma_{xy}(\omega) = a(\omega)\frac{-v_F^2 \mathbf{q}_\perp^2}{\omega^2 - v_F^2 \mathbf{q}_\perp^2} + b(\omega)\frac{\omega^2}{\omega^2 - v_F^2 \mathbf{q}_\perp^2}$$

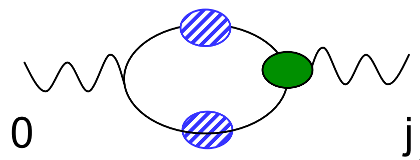
Goldstone mode

$$\mathbf{q}_\perp = (q_x, q_y, 0)$$

NOTE) electrons are 2D-like

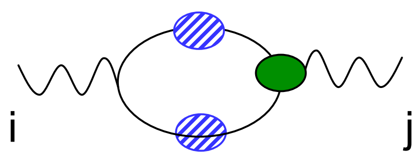
a(ω) and b(ω) come from Fermion loops like…

$$a(\omega) = \frac{1}{2}\varepsilon_{ij}\frac{\partial}{\partial q_i}\left\{ \cdots \right\}\Bigg|_{\boldsymbol{q}=0}$$



density-current correlation

$$b(\omega) = \frac{1}{2i\omega}\varepsilon_{ij}\left\{ \cdots \right\}$$

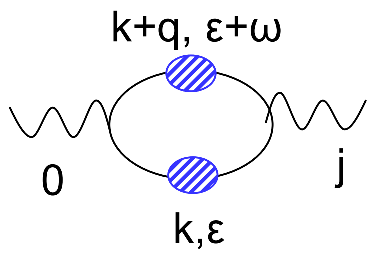


current-current correlation

Because of the spontaneous gauge symmetry breaking,

$$a(\omega) \neq b(\omega)$$

$$a(\omega) = \frac{1}{2}\varepsilon_{ij}\frac{\partial}{\partial q_i}\left\{ \cdots \right\}\Bigg|_{\mathbf{q}=0} + \text{vertex corrections}$$



density-current correlation

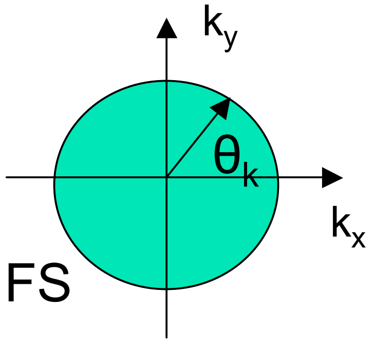
Cf) ・ <u>dc-limit;</u> Volovik ('88), JG-Ishikawa('98), Read&Green('00), Furusaki, Matsumoto & Sigrist ('01), Holovitz-Galub('02), Stone('02)

・ <u>ac-limit;</u> Yakovenko('06), Lutchyn et al ('08), Roy & Kalline ('08))

- induced ***WITHOUT vertex corrections***
- related to the ***TOPOLOGY of the gap function in momentum space***

Chiral *p* Chiral *d*

Chiral SCs (L_z= ±1, ±2…); $\Delta_{\mathbf{k}} = \Delta \exp[iL_z\theta_{\mathbf{k}}]$



$$a(\omega) \propto \frac{1}{|\Delta_{\mathbf{k}}|^2}\oint_{FS}\frac{d\theta_{\mathbf{k}}}{2\pi i}\Delta^*_{\mathbf{k}}\frac{d}{d\theta_{\mathbf{k}}}\Delta_{\mathbf{k}} = \underline{L_z}$$

winding #

… analogous to the INTRINSIC part of AHE, where σ_{xy} is given by TKNN Chern # (Kaplas-Luttinger, Nagaosa et al)

For chiral p-wave (L_z=1) …

(i) T=0,

(ii) without vertex correction

$$a(\omega) = \frac{e^2}{4\pi d} \begin{cases} \frac{i\pi\Delta^2}{2\omega^2} - \frac{\Delta^2}{\omega^2}\ln(\frac{\omega}{\Delta}) & \text{for } |\Delta| < \omega << \omega_D \\ -\frac{|\Delta|^2}{\omega^2}\ln(\frac{\omega_D}{|\Delta|}) & \text{for } \omega_D << \omega \end{cases}$$

ω_D; cut-off energy of pairing interaction

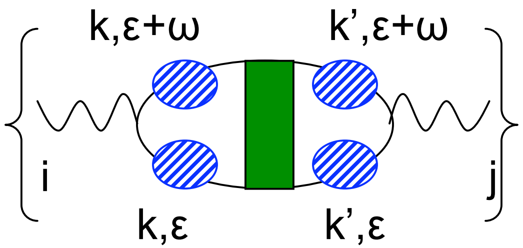
d; interlayer distance

R.Roy and C. Kalline, PRB ('08)
See, also, Lutchyn, Nagornykh, and Yakovenko ('08)

It has been checked that …

- In GL region T∽Tc, ω-dependence is the same essentially
- Vertex corrections give higher order contributions of $1/\omega$

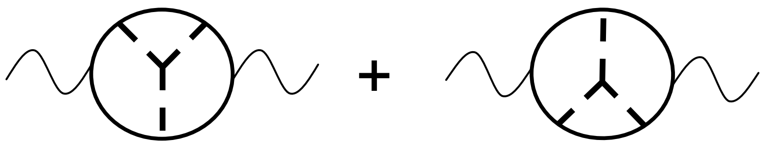
$$b(\omega) = \frac{1}{2i\omega}\varepsilon_{ij}$$



current-current correlation

- ***induced by vertex correction***

  The lowest-order contribution; J.G. PRB ('08)



Cf) skew-scattering of the EXTRINSIC AHE and SHE
Smit ('58)

- $L_z \neq 0$ is necessary

- but ***NOT*** proportional to L_z

ex) point-like (s-wave) scattering channel;

chiral p-wave (L_z=*1*) state→*nonzero*
Higher-chiral states (L_z=2,3,4…)→*0*

this result would be sensitive to scattering channel

For chiral p-wave (L_z=1) …

- T∽Tc
- in the dilute limit of point-like impurities

$$b(\omega) = \frac{e^2}{4\pi d} \frac{|\Delta^2|\epsilon_{\rm F}}{\pi} \sin 2\delta_0 \times \begin{cases} \frac{-i4\pi^2\Gamma}{\omega(\omega+i\Gamma)^3} & (T_c << \omega << \omega_D) \\ \frac{-i32\omega_D^2\Gamma}{\omega(\omega+i\Gamma)^5} & (\omega_D << \omega) \end{cases}$$

δ_0; phase shift

ε_F; Fermi energy

Γ; quasiparticle dumping

ω_D; cut-off energy of pairing interaction

d; interlayer distance

III. Kerr angle

$$\theta_K = -Im\left[\frac{\omega(q_z^+ - q_z^-)}{\omega^2 - q_z^+ q_z^-}\right]$$

$$q_z^\pm = \sqrt{\omega^2 + i\omega\sigma_{xx} \pm \omega\sigma_{xy}}$$

σ_{xx} ? …in the measurement of PKE,

$\omega \backsim 0.8$ eV >> $2|\Delta|$, q << $1/\xi$

(high frequency and long wavelength)

→ Drude formula (verified experimentally by Katsufuji et al JPSJ ('94))

$$\sigma_{xx}(\omega) = \frac{\omega_p^2 \tau_0}{1 - i\omega\tau_0}$$

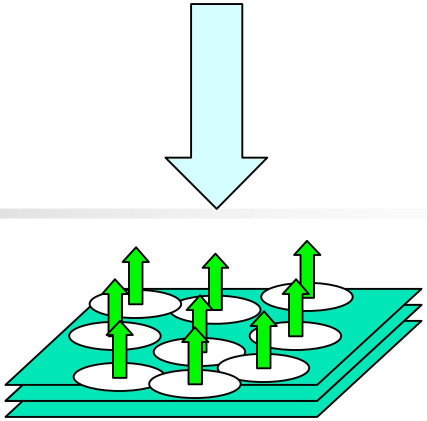
ω_p; plasma frequency

τ_0; quasiparticle life time

○ WITHOUT extrinsic effect

i.e. neglect the vertex corrections

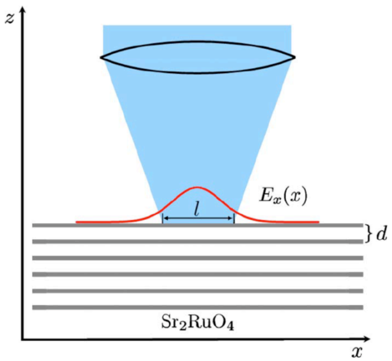
Luchyn et al ('08), Roy-Kallin ('08)



$$\sigma_{xy}(\omega) = a(\omega)\frac{-v_F^2 \mathbf{q}_\perp^2}{\omega^2 - v_F^2 \mathbf{q}_\perp^2}$$

$$\mathbf{q}_\perp = (q_x, q_y, 0)$$

• In the experiment, **q** // z…



if q is *completely* parallel to z-axes,
$\sigma_{xy}=0$ ∴$\theta_K=0$

・・・ q_x,q_y-components caused by lens focus effect can be taken into account, but σ_{xy}<<1and

→ $\theta_K \backsim 10^{-17}$ rad << 10^{-9} rad (obs. value)

even if in a optimized condition $\omega_D \to \infty$

○ INCLUDING the Extrinsic part *b(ω)*

Cf) in 2nd Born; J.G. PRB ('08)

$$\sigma_{xy}(\omega) = a(\omega)\frac{-v_{\rm F}^2\mathbf{q}_\perp^2}{\omega^2 - v_{\rm F}^2\mathbf{q}_\perp^2} + b(\omega)\frac{\omega^2}{\omega^2 - v_{\rm F}^2\mathbf{q}_\perp^2}$$

$$\mathbf{q}_\perp = (q_x, q_y, 0)$$

- for simplicity, ***q*** // z → $\sigma_{xy}(\omega) = b(\omega)$

Realistic orders of parameters

$T_c = 10^{-4}$ eV,

$d = 6.5\,\mathring{A}$, $\varepsilon_F = 0.14$ eV,

$\Gamma = 6.6\text{x}10^{-5}$eV, $\omega_p = 1.3$eV

$\omega = 0.8$ eV

Unknown parameters;

cut off; $\omega_D = 1000T_c \backsim \infty$

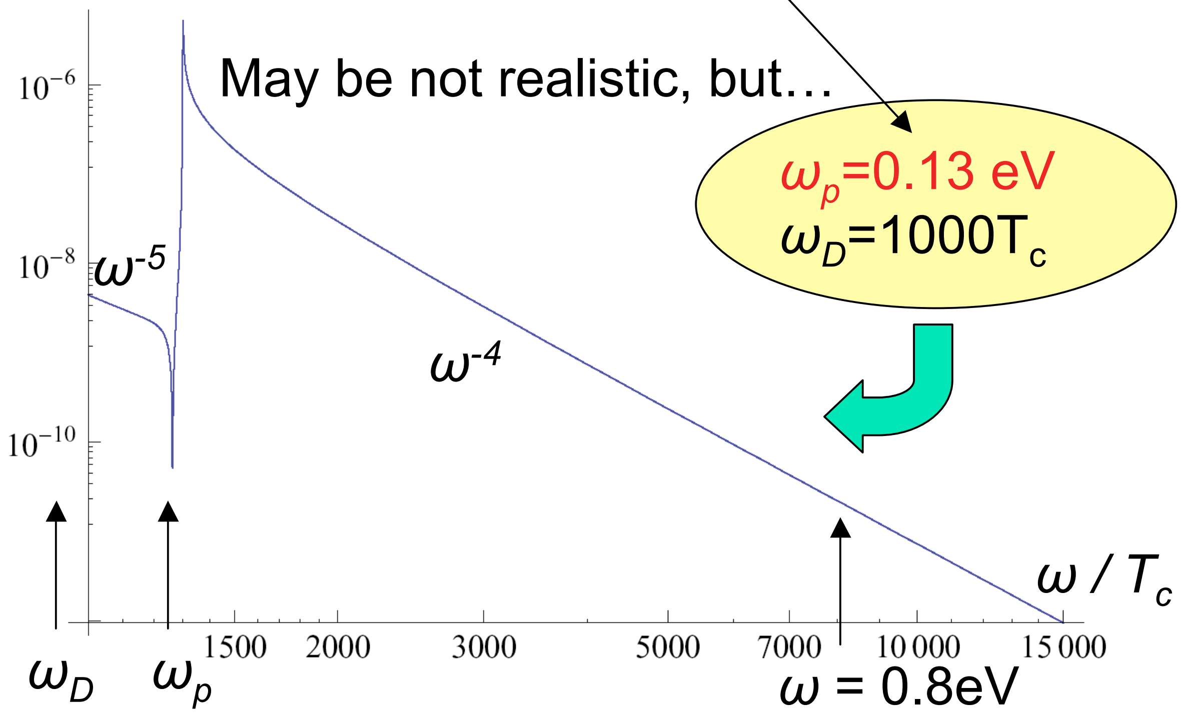
phase shift; $\delta_0 = 0.1\pi$

$\theta_K \backsim 10^{-16} \backsim 10^{-14} << 10^{-9}$ (Obs.)

much smaller…

May be not realistic, but…

ω_p=0.13 eV
ω_D=1000T$_c$

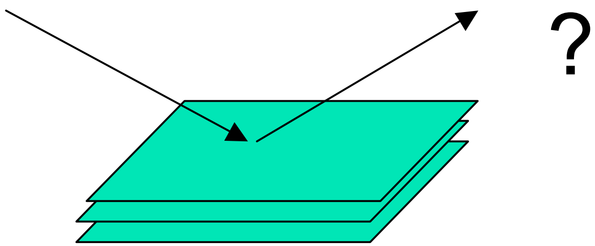


Anyway, experiment in low frequency region is highly desired.

How to enhance a signal from the intrinsic part a(ω) $\propto L_z$?

$$\sigma_{xy}(\omega) = a(\omega)\frac{-v_F^2\mathbf{q}_\perp^2}{\omega^2 - v_F^2\mathbf{q}_\perp^2} + b(\omega)\frac{\omega^2}{\omega^2 - v_F^2\mathbf{q}_\perp^2}$$

- Finite $\mathbf{q}_\perp = (q_x, q_y, 0)$ is needed …



- But it should be noted that…

$$\frac{\text{intrinsic}}{\text{extrinsic}} = -\frac{a(\omega)}{b(\omega)}\frac{v_F^2 q_\perp^2}{\omega^2} \quad , \quad v_F^2 \ll 1$$

… then, the extrinsic part seems to be dominant in most cases

Summary

"Zero field Hall conductivity" in chiral p-wave state

- intrinsic and extrinsic origins

- application to polar Kerr effect ***q***//z

  extrinsic part b(ω) is dominant

- how to see the intrinsic part a(ω) ?

For quantitative agreement…
Multiband effect?