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@Sapporo Winter School, Hokkaido university

Toward a Proof of Montonen-Olive Duality via Multiple M2-branes

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BLG theory / ABJM theory

BLG: Bagger-Lambert-Gustavsson (0711.0995,0709.1260)
ABJM : Aharony-Bergman-Jafferis-Maldacena (0806.1218)

= Theory describing multiple M2-branes in M-theory

= 2+1 dimensional non-Abelian Chern-Simons theory

M-theory : 11 dimensional theory which describes
strong coupling limit of type IIA superstring

M2-branes : fundamental objects of the M-theory

Theory of multiple M2-branes → definition of M-theory?!

Forget the importance of M-theory.

Is BLG / ABJM useful for any other purpose?

Ans: Surely it is very useful, for non-Abelian duliaty

Montonen-Olive duality conjecture

Proof wanted !!!

4d $\mathcal{N}=4$ *U(N)* super Yang-Mills

$$S = \frac{-1}{8\pi}\int d^4x \, \mathrm{tr}\left[\mathrm{Im}(\tau) F_{MN}F^{MN} + \mathrm{Re}(\tau)\frac{1}{2}\epsilon^{MNPQ}F_{MN}F_{PQ}\right]$$

$$\tau \equiv \frac{\theta}{2\pi} + \frac{4\pi i}{g_{\mathrm{YM}}^2}$$

has a $SL(2,\mathbf{Z})$ non-local symmetry of the theory,

$$\tau' = \frac{a\tau+b}{c\tau+d}, \quad ad-bc=1, \quad a,b,c,d \in \mathbf{Z}$$

Non-Abelian dualities ⇔ String/M-theory

M-theory → (S[1] w/ R_{11}) → IIA string w/ $g_s^{(A)}$ → (S[1] w/ R_9) → IIA string on S[1]

M2 — D2 — D2

IIA string on S[1] ⇕ T-dual ⇕ IIB string on S[1]

IIB string w/ $g_s^{(B)}$ → (S[1] w/ R) → IIB string on S[1]

D3 — D3

M2 ⇔ D2 : Scalar-Vector non-Abelian duality $\Phi^{11} \leftrightarrow A_\mu$

[Mukhi,Papageorgakis(08)] [Distler,Mukhi,Papageorgakis,VanRaamsdonk(08)]

D3 ⇔ D3 : MO duality = “9-11 flip” in M-theory

Decompactified IIB = M on shrinking torus

$$g_s^{(B)} = R_{11}/R_9 \qquad R = l_P^3/(R_{11}R_9)$$

Strategy for a field theory proof of MO

ABJM = theory of multiple M2-branes

Realize a torus compactification of transverse space for ABJM

Shrinking torus limit

"9-11 flip"

··· = 4d YM = 4d YM = 4d YM = ···

MO duality

Question 1 : Torus in ABJM ?

Ans : Quiver generalization of ABJM + 2 vevs

Question 2 : Choice of the torus cycles ?

Ans : Different pairings of CS gauge fields

ABJM theory and S^1 compactification

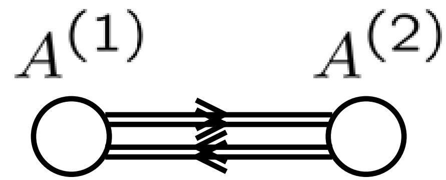
U(N) x U(N) ABJM theory = *N* M2s on **C**^{4} / **Z**$_k$

[Aharony,Bergman,Jafferis,Maldacena]

$$S=\int d^3x\left[\frac{k}{4\pi}\epsilon^{\mu\nu\lambda}\mathrm{tr}\left(A^{(1)}_\mu\partial_\nu A^{(1)}_\lambda+\frac{2i}{3}A^{(1)}_\mu A^{(1)}_\nu A^{(1)}_\lambda-A^{(2)}_\mu\partial_\nu A^{(2)}_\lambda-\frac{2i}{3}A^{(2)}_\mu A^{(2)}_\nu A^{(2)}_\lambda\right)\right.$$
$$\left.-\mathrm{tr}\left((D_\mu Z_A)^\dagger D^\mu Z^A\right)-\mathrm{tr}\left((D_\mu W^A)^\dagger D^\mu W_A\right)-V(Z,W)\right] \qquad A=1,2$$

$$D_\mu Z^A=\partial_\mu Z^A+iA^{(1)}_\mu Z^A-iZ^A A^{(2)}_\mu,\quad D_\mu W^A=\partial_\mu W^A+iA^{(2)}_\mu W^A-iW^A A^{(1)}_\mu$$

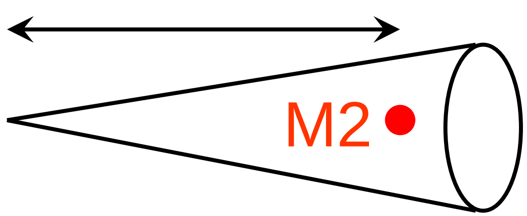
Quiver diagram of ABJM



S^1 from Orbifold :

[Distler,Mukhi,Papageorgakis,VanRaamsdonk]
[Honma,Iso,Sumitomo,Zhang]

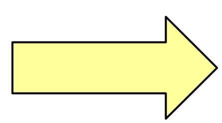
$$Z^1=v\mathbf{1}_{N\times N}$$



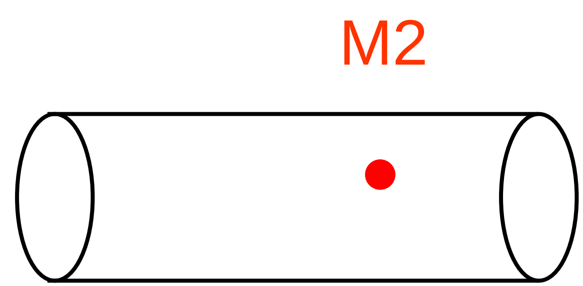
Orbifold **C**^{4} / **Z**$_k$

$v\to\infty$

$k\to\infty$



v/k : fixed

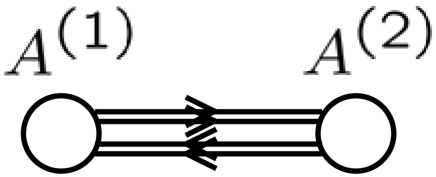


S^1 compactification

ABJM → 3d YM

[Mukhi,Papageorgakis] [Distler,Mukhi,Papageorgakis,VanRaamsdonk]

Linear combination : $A_\mu^{(\pm)} \equiv \frac{1}{2}\left(A_\mu^{(1)} \pm A_\mu^{(2)}\right)$



Vev v gives the mass term for $A^{(-)}$

$$\int d^3x \frac{k}{2\pi}\epsilon^{\mu\nu\lambda}\mathrm{tr}\left[A_\mu^{(-)}F_{\nu\lambda}^{(+)} + \frac{2i}{3}A_\mu^{(-)}A_\nu^{(-)}A_\lambda^{(-)}\right] - \int d^3x\ \mathrm{tr}\left[\{A_\mu^{(-)}, v\}^2\right]$$

The massive $A^{(-)}$ is an auxiliary field, integrated out

$$A_\mu^{(-)} = \frac{k}{16\pi v^2}\epsilon_{\mu\nu\lambda}F^{(+)\nu\lambda}$$

$v \to \infty$ $k \to \infty$

v/k : fixed

3d YM $$S = -\int d^3x \frac{k^2}{32\pi^2 v^2}\mathrm{tr}\left[(F_{\mu\nu}^{(+)})^2 + \frac{k^4}{v^6}\mathcal{O}((F^{(+)})^3)\right]$$

ABJM in the limit = 3d *U(N)* YM (*N* D2-brane action)

Scalar kinetic term for $\mathrm{Im}\ Z^1$ dissapears.

ABJM → 4d YM

Another circle ?

We need a torus …… Another orbifold is necessary

ABJM → Orbifolded $U(N)^n \times U(N)^n$ ABJM

Douglas-Moore

[Bena,Klebanov,Klose,Smedback] [Terashima,Yagi] [Imamura,Kimura] [Hosomichi, Lee[3],Park]

$$S = \int d^3x \left[\frac{k}{4\pi}\epsilon^{\mu\nu\lambda} \sum_{l=1}^{n} \mathrm{tr}\left(A_\mu^{(2l-1)}\partial_\nu A_\lambda^{(2l-1)} + \frac{2i}{3}A_\mu^{(2l-1)}A_\nu^{(2l-1)}A_\lambda^{(2l-1)} - A_\mu^{(2l)}\partial_\nu A_\lambda^{(2l)} - \frac{2i}{3}A_\mu^{(2l)}A_\nu^{(2l)}A_\lambda^{(2l)} \right) - \mathrm{tr}\sum_{s=1}^{2n}\left((D_\mu Z^{(s)})^\dagger D^\mu Z^{(s)} + (D_\mu W^{(s)})^\dagger D^\mu W^{(s)} \right) - V(Z,W) \right]$$

$$D_\mu Z^{(2l-1)} = \partial_\mu Z^{(2l-1)} + iA_\mu^{(2l-1)}Z^{(2l-1)} - iZ^{(2l-1)}A_\mu^{(2l)},$$

$$\bullet\bullet\bullet \Rightarrow D_\mu Z^{(2l)} = \partial_\mu Z^{(2l)} + iA_\mu^{(2l)}Z^{(2l)} - iZ^{(2l)}A_\mu^{(2l+1)}, \quad \bullet$$

$$D_\mu W^{(2l-1)} = \partial_\mu W^{(2l-1)} + iA_\mu^{(2l)}W^{(2l-1)} - iW^{(2l-1)}A_\mu^{(2l-1)},$$

$$D_\mu W^{(2l)} = \partial_\mu W^{(2l)} + iA_\mu^{(2l+1)}W^{(2l)} - iW^{(2l)}A_\mu^{(2l)}.$$

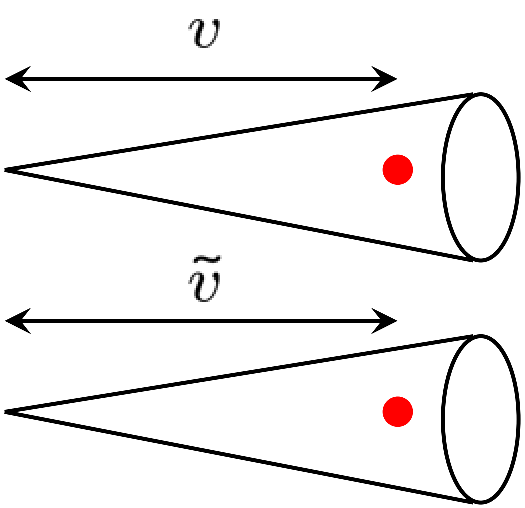
Torus compactification

Moduli space : $\mathbf{C}^4/(\mathbf{Z}_n \times \mathbf{Z}_{nk})$ [Terashima,Yagi] [Imamura,Kimura]

$$(z_1, w_1, z_2, w_2) \sim (e^{2\pi i/kn} z_1, e^{-2\pi i/kn} w_1, e^{-2\pi i/kn} z_2, e^{2\pi i/kn} w_2)$$
$$\sim (z_1, w_1, e^{2\pi i/n} z_2, e^{-2\pi i/n} w_2).$$

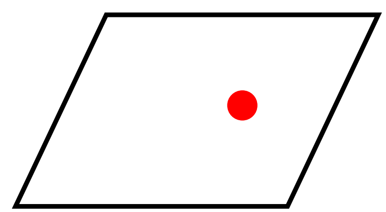
It's already a double orbifold, so we take $k = 1$

Turn on two vevs :
$$Z^{(2l-1)} = v,\ Z^{(2l)} = \tilde{v}$$
$$W^{(2l-1)} = W^{(2l)} = 0$$



$$v,\ \tilde{v},\ n \to \infty,$$
$$v/n,\ \tilde{v}/n \to 0,$$

$v/\tilde{v}$: fixed

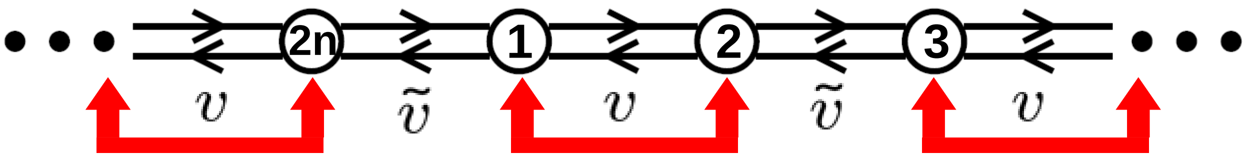


M2 on a shrinking torus

1st step : integration out

For simplicity in this talk I will explain the case $v \gg \tilde{v}$

Linear combination : $A_\mu^{(\pm)(2l-1)} \equiv \frac{1}{2}\left(A_\mu^{(2l-1)} \pm A_\mu^{(2l)}\right)$



$$S = \int d^3x \sum_{l=1}^{n} \mathrm{tr}\left[\frac{1}{2\pi}\epsilon^{\mu\nu\lambda}\left(A_\mu^{(-)(2l-1)} F_{\nu\lambda}^{(+)(2l-1)}\right) - 4v^2 (A_\mu^{(-)(2l-1)})^2 - \tilde{v}^2 \left(A_\mu^{(+)(2l-1)} - A_\mu^{(+)(2l+1)}\right)^2\right]$$

Integrating out the auxiliary fields $A_\mu^{(2l-1)(-)}$

$$S = \int d^3x \ \mathrm{tr}\left[-\frac{1}{32\pi^2 v^2}\sum_{l=1}^{n}\left(F_{\mu\nu}^{(+)(2l-1)}\right)^2 + \sum_{l,l'=1}^{n} A_\mu^{(+)(2l-1)} M_{ll'} A^{(+)(2l'-1)\mu}\right]$$

2nd step : KK summation

$$S = \int d^3x \; \mathrm{tr}\left[-\frac{1}{32\pi^2 v^2} \sum_{l=1}^{n} \left(F_{\mu\nu}^{(+)(2l-1)}\right)^2 + \sum_{l,l'=1}^{n} A_\mu^{(+)(2l-1)} M_{ll'} A^{(+)(2l'-1)\mu} \right]$$

$$M = -v^2 \begin{pmatrix} 2 & -1 & 0 & \cdots & \\ -1 & 2 & -1 & 0 & \cdots \\ 0 & -1 & 2 & -1 & \cdots \\ 0 & 0 & -1 & 2 & \cdots \\ \cdots & & & & \end{pmatrix}$$

→ Diagonalize.

$$\lambda_l = 2 - 2\cos(2\pi l/n)$$

In the limit $n \to \infty$,

$$\lambda = (2\pi s/n)^2 \quad (s \in \mathbf{Z})$$

$$S = \int d^3x \; \mathrm{tr}\left[-\frac{n}{32\pi^2 v^2} \hat{\mathcal{L}}_{\mathrm{kin}} - 4\tilde{v}^2 n \sum_{s \in \mathbf{Z}} \left(\frac{s\pi}{n}\right)^2 (\hat{A}_\mu^{(+)(s)})^2 \right]$$

$$\hat{\mathcal{L}}_{\mathrm{kin}} \equiv \sum_s (\partial_\mu \hat{A}_\nu^{(+)(s)} - \partial_\nu \hat{A}_\mu^{(+)(s)})^2 + \cdots$$

⇔ equivalent

$$S = -\frac{\tilde{v}}{8\pi v} \int d^4x \; \mathrm{tr}\left[F_{MN}^2\right]$$

4d YM

KK radius

$$\frac{1}{R} = \frac{8\pi^2 v \tilde{v}}{n}$$

Final 4d action

We obtain 4d *N=4* SYM action from the orbifolded ABJM,

$$S = \int d^4x \, \mathrm{tr} \left[-\frac{v\tilde{v}}{8\pi(v^2+\tilde{v}^2)} F_{MN}^2 + \frac{\tilde{v}^2}{16\pi(v^2+\tilde{v}^2)} \epsilon^{MNPQ} F_{MN} F_{PQ} \right]$$

Theta term appears.

For generic $v, \tilde{v}$, cross terms remain → theta

"3d → 4d" is ….
- Deconstruction. [Arkani-Hamed,Cohen,Georgi] [Hill,Pokorski,Wang]
- Taylor's T-duality. [Taylor]

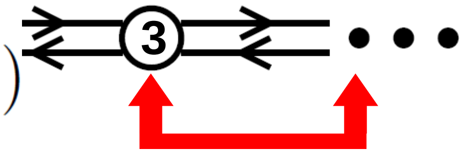
SUSY enhancement : 3d *N=4* → 4d *N=4* in the limit

MO duality

Choice of the combination

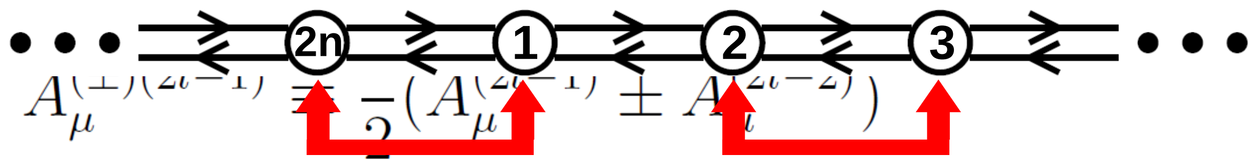
Prime case

- $$A_\mu^{(\pm)(2l-1)} \equiv \frac{1}{2}\left(A_\mu^{(2l-1)} \pm A_\mu^{(2l)}\right)$$



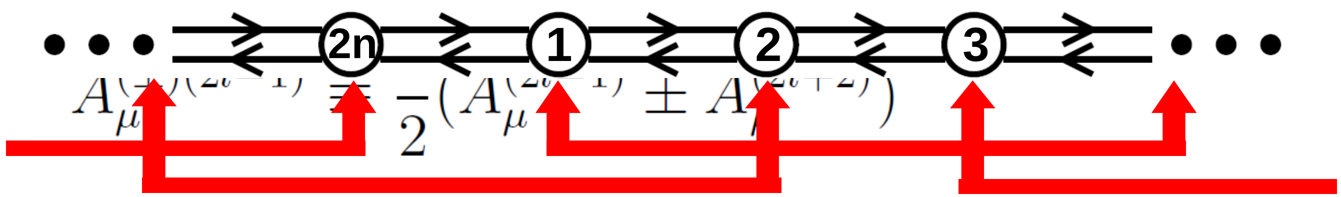
$$\tau = \frac{-\ \tilde{v}^2}{v^2+\tilde{v}^2} + i\frac{v\tilde{v}}{v^2+\tilde{v}^2}$$

Second case



$$\tau' = \frac{-\ v^2}{v^2+\tilde{v}^2} + i\frac{v\tilde{v}}{v^2+\tilde{v}^2}$$

Third case



$$\tau' = \frac{-\ (v^2+2\tilde{v}^2)}{v^2+\tilde{v}^2} + i\frac{v\tilde{v}}{v^2+\tilde{v}^2}$$

Emergence of SL(2,Z)

These couplings are related by SL(2,Z) transformation!

$$\tau' = \frac{a\tau + b}{c\tau + d}, \quad ad - bc = 1, \quad a, b, c, d \in \mathbf{Z}$$

Generators : $T(\tau) = \tau + 1 \qquad S(\tau) = -\tau^{-1}$

Prime → Second $\quad \tau' = S(T^2(S(T(\tau))))$

Prime → Third $\quad \tau' = T^{-1}(\tau)$

SL(2,Z) transformation = Choice of combination of CS gauge fields

MO duality proven ?

Is this a proof of MO duality ?!?!

Not really…..

We find   Our S transf. = Parity transf.

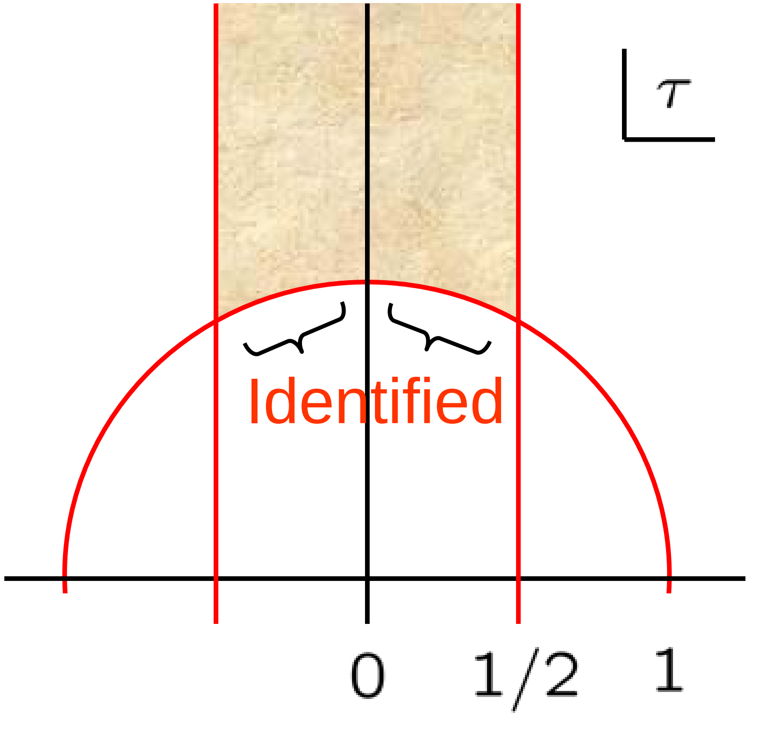
Our coupling is a one-parameter family

$$\tau = \frac{-\tilde{v}^2}{v^2+\tilde{v}^2} + i\frac{v\tilde{v}}{v^2+\tilde{v}^2}$$

$$\tau' = \frac{-v^2}{v^2+\tilde{v}^2} + i\frac{v\tilde{v}}{v^2+\tilde{v}^2}$$

$$= S(T^2(S(T(\tau))))$$

$$= P(T(\tau))$$



Summary : importance

Torus compactification of M2 transverse space is realized.

Orbifolded ABJM + 2 vevs, with a new scaling limit

4d YM on N D3s is derived from 3d ABJM.

Deconstruction (T-duality) with theta term

Infinitely many 4d YM are derived.

New geometric interpretation of quiver diagrams

Montonen-Olive duality is proven?

No. Our S is a parity, due to our special values of τ.

However, field theory realization of T transf. is nontrivial

We got half the way to the goal.

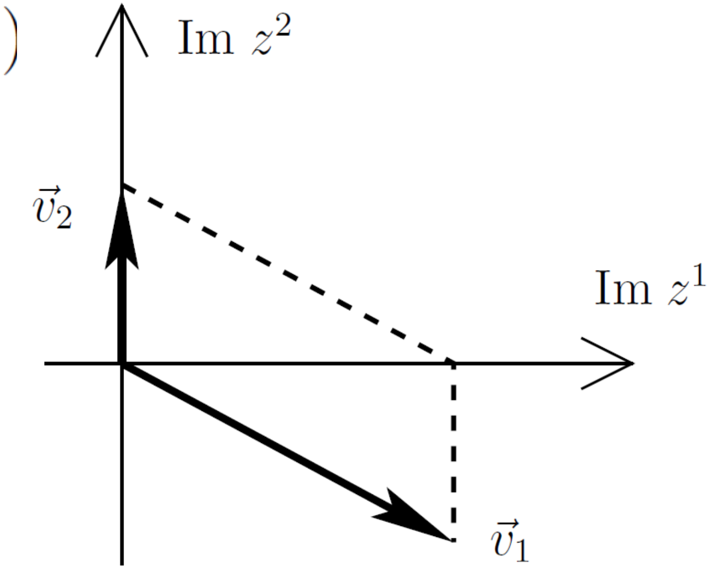
M-theory interpretation

$$(z_1, w_1, z_2, w_2) \sim (e^{2\pi i/kn} z_1, e^{-2\pi i/kn} w_1, e^{-2\pi i/kn} z_2, e^{2\pi i/kn} w_2)$$
$$\sim (z_1, w_1, e^{2\pi i/n} z_2, e^{-2\pi i/n} w_2).$$

2 vevs $(z_1, w_1, z_2, w_2) = (v, 0, \tilde{v}, 0)$

→ 2 1-cycles specified by vectors

$$\begin{cases} \vec{v}_1 \equiv (2\pi i v/kn, 0, -2\pi i \tilde{v}/kn, 0) \\ \vec{v}_2 \equiv (0, 0, 2\pi i \tilde{v}/n, 0) \end{cases}$$



Torus modurus

⇨ $$\tau = \frac{-k\tilde{v}^2}{v^2 + \tilde{v}^2} + i \frac{kv\tilde{v}}{v^2 + \tilde{v}^2}$$

Due to M-IIB duality, this should become axio-dilaton.

Our field theory result is identical with this

Discussions

Toward a Proof ?

Other vevs don't help for our quiver…..

→ Different quivers necessary?

IIB physics ⇔ M physics?

Interpolation of AdS_5 ⇔ AdS_4 ?