Aharonov-Bohm Effect
and
Gauge-Higgs Unification

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YH, Oda, Ohnuma, Sakamura, PRD 78 (2008) 096002 (arXive: 0806.0480 [hep-ph])
YH, Y.Kobayashi, arXive: 0812.4782 [hep-ph]

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Congratulations

還暦

Ishikawa-san, Kawamoto-san

field theory, topology, physics

Why is the Higgs field necessary?

GUT Higgs

Grand Unified Theory (GUT)
$SU(5)$

→

Strong int.
$SU(3)_C$

Electroweak Unified Theory
$SU(2)_L \times U(1)_Y$

EW Higgs

→

Weak int.

EM int.
$U(1)_{EM}$

Gauge unification
Large gauge sym.

⟺

Mechanism of sym. breaking
Higgs field

In the standard model

EW Higgs

Electroweak Unified Theory
$SU(2)_L \times U(1)_Y$

→ *Weak int.*

→ *EM int.* $U(1)_{EM}$

Gauge sector : established.

Higgs sector : yet to be unveiled.

No principle

Many free parameters

Unnatural against radiative corrections

Hierarchy in fermion masses

Gauge-Higgs Unification

A_M in higher dimensions

4-dim. components A_μ

$\cup$

4D gauge fields

extra-dim. component A_y

$\cup$

4D Higgs fields
(Aharonov-Bohm phase)

Fairlie **(1979)** *Manton* **(1979)**

on $M^4 \times S^2$ $\quad SO(5), SU(3), G_2 \to SU(2)_L \times U(1)_Y$

Assumptions: **Spherical configurations**
Non-zero flux

$m_{KK} \sim m_Z, m_H$

⇨

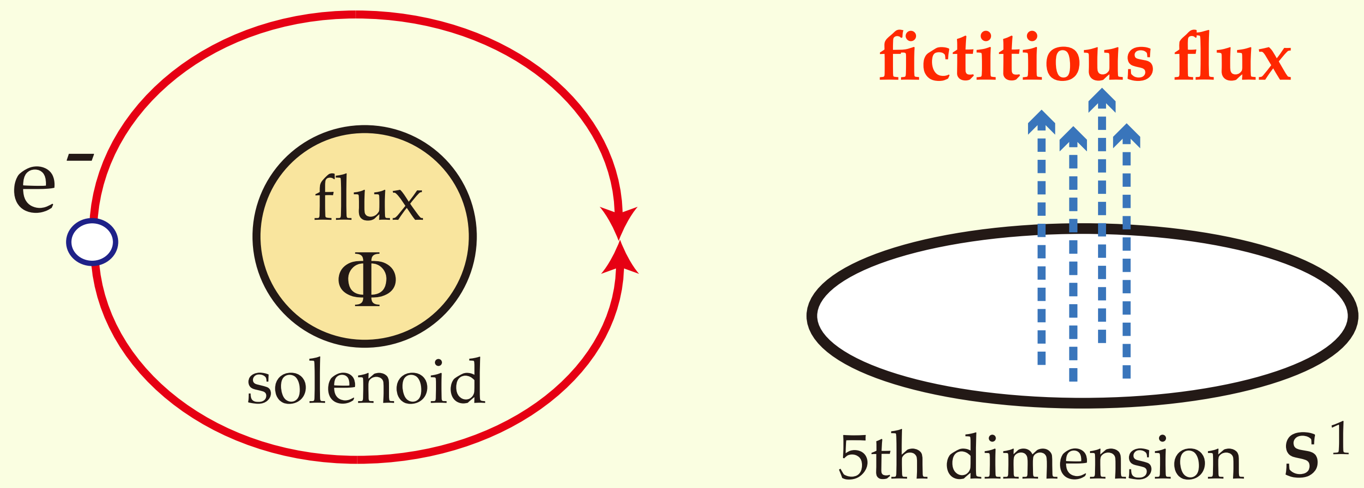
G-H unification on
non-simply-connected space

4D Higgs = AB phases in extra dimensions

Dynamical gauge symmetry breaking

Hosotani 1983

Aharonov-Bohm Effect



Relevant quantity is

AB phase

$$\exp\left\{ ig \int A_y dy \right\} \sim e^{i\Theta_H(x)}$$

Higgs field

AB phase θ_H

$$S^1 : \quad E_n(\vec{p})^2 = m^2 + \vec{p}^{\,2} + \frac{1}{R^2}\left(n - \frac{\theta_H}{2\pi}\right)^2$$

$$V_{\rm eff}(\theta_H) = \sum \int \frac{d^3p}{(2\pi)^3} \sum_n \pm \frac{1}{2} E_n(\vec{p}\,;\theta_H)$$

- finite

$m_H^2 \propto V_{\rm eff}^{(2)}$: finite

non-Abelian :

θ_H : matrix $\rightarrow$ gauge sym. breaking

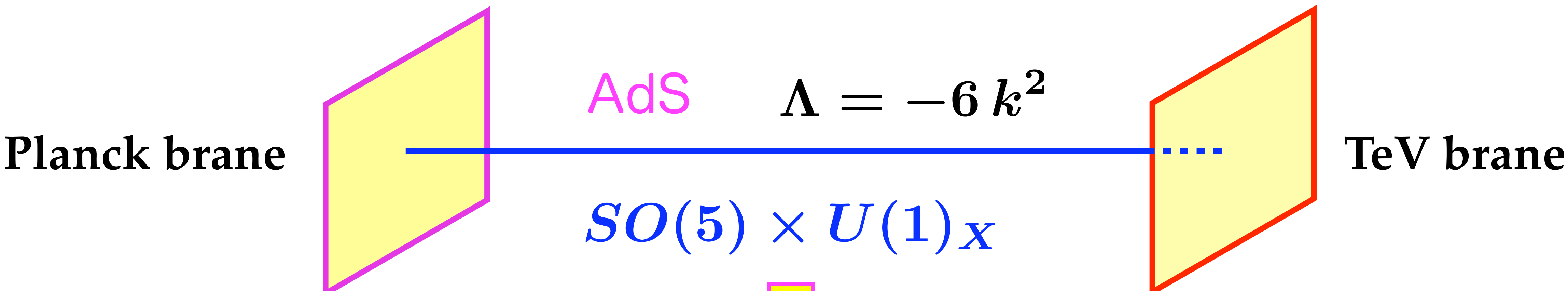
and more

EW Gauge-Higgs Unification in RS

Randall-Sundrum warped space

$$ds^2 = e^{-2k|y|} dx_\mu dx^\mu + dy^2$$

$$0 \le |y| \le L = \pi R$$



orbifold ⇩

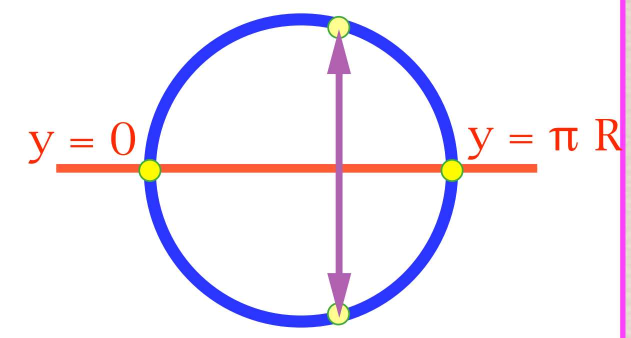
$SU(2)_L \times U(1)_Y$

⇩ *AB effect*
Hosotani mechanism

$U(1)_{EM}$

Orbifolds

Pomarol, Quiros 1998



$M^4 \times (S^1/Z_2)$

Randall-Sundrum

Identify y **&** $-y$ ⇨ $\mathcal{L} = \frac{1}{2} \mathrm{Tr}\, {F_{MN}}^2$: invariant

$$\begin{pmatrix} A_\mu \\ A_y \end{pmatrix} (x, -y) = P_0 \begin{pmatrix} A_\mu \\ -A_y \end{pmatrix} (x, y) P_0^\dagger$$

$$\begin{pmatrix} A_\mu \\ A_y \end{pmatrix} (x, \pi R - y) = P_1 \begin{pmatrix} A_\mu \\ -A_y \end{pmatrix} (x, \pi R + y) P_1^\dagger$$

$$F_{\mu y} = \partial_\mu A_y - \partial_y A_\mu + ig[A_\mu, A_y]$$

⇨ *Orbifold BC :* P_0 , P_1

Origin of the Higgs doublet

$SO(5) \times U(1)_X$

Agashe, Contino, Pomarol 2005
Hosotani, Sakamura 2006
Medina, Shah, Wagner 2007

$$P_0 = P_1 = \begin{pmatrix} -1 & & & & \\ & -1 & & & \\ & & -1 & & \\ & & & -1 & \\ & & & & +1 \end{pmatrix}$$

$SO(5) \to SO(4) \simeq SU(2)_L \times SU(2)_R$

W Z γ

$$A_\mu \sim \begin{pmatrix} \boxed{} & \\ & \end{pmatrix}$$

Higgs

$$A_y \sim \begin{pmatrix} & \begin{matrix}\phi_1\\ \phi_2\\ \phi_3\\ \phi_4\end{matrix} \\ \boxed{} & \end{pmatrix} \qquad \Phi = \begin{bmatrix} \phi_1 + i\phi_2 \\ \phi_4 - i\phi_3 \end{bmatrix}$$

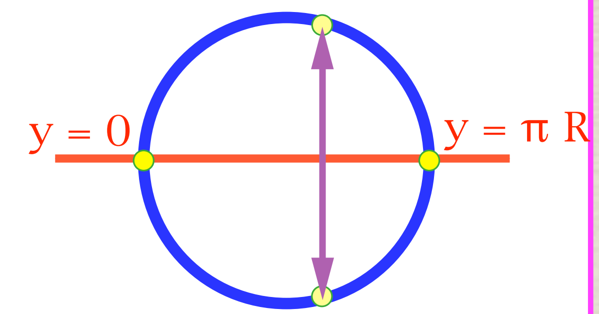
$$\theta_H \sim g \int dy\, A_y$$

Wilson line

= 4D Higgs

Chiral fermions

$$\psi(x, -y) = P_0 \gamma^5 \psi(x, +y)$$
$$\psi(x, \pi R - y) = P_1 \gamma^5 \psi(x, \pi R + y)$$



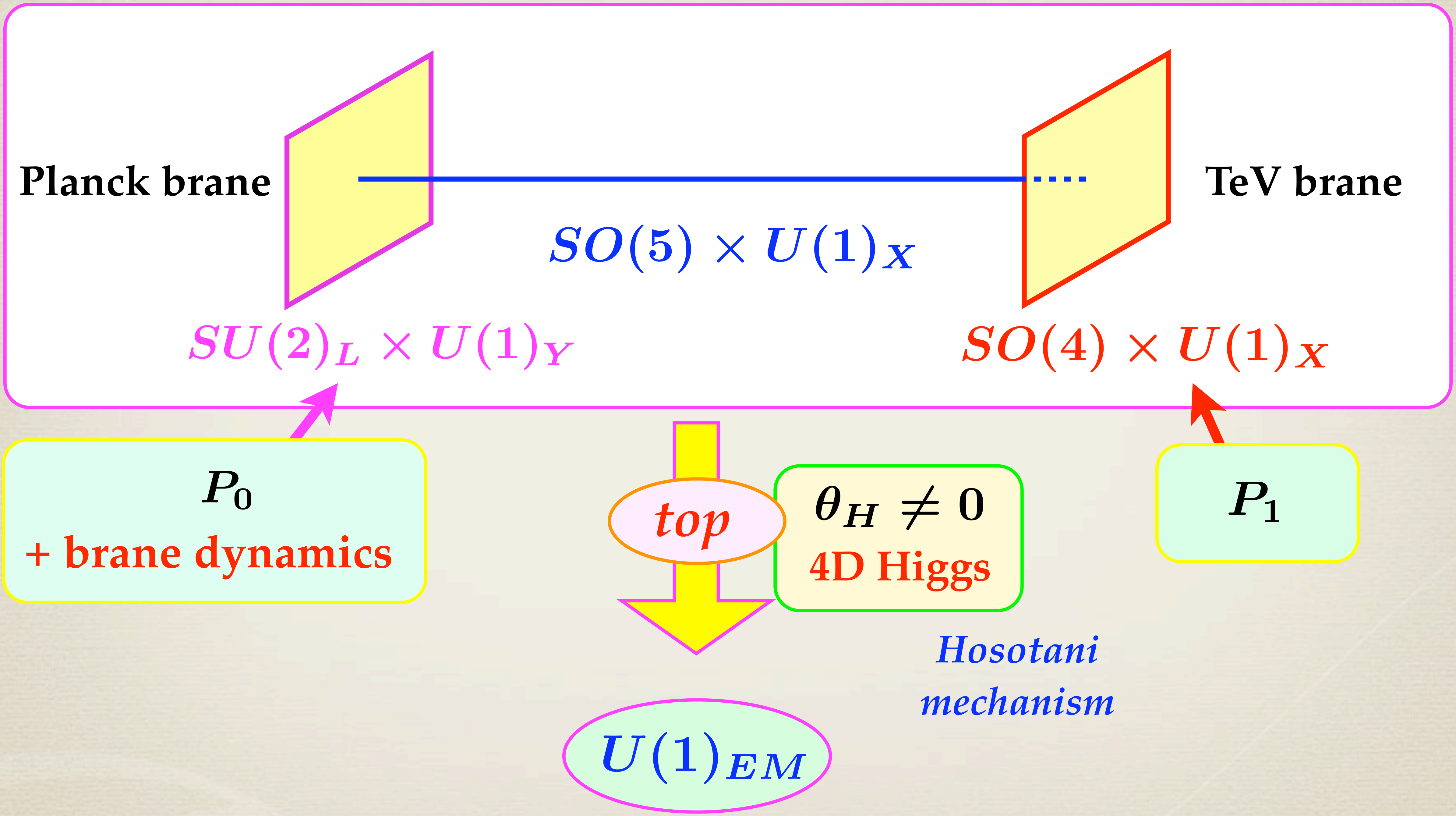
$$\bar{\psi}\{ i\gamma^\mu D_\mu + i\gamma^5 (\partial_y - i g A_y)\}\psi$$

$$P_0 = P_1 = \begin{pmatrix} -1 & & & & \\ & -1 & & & \\ & & -1 & & \\ & & & -1 & \\ & & & & +1 \end{pmatrix}$$

$$\begin{pmatrix} T \\ B \\ t \\ b \\ t' \end{pmatrix}_{\frac{2}{3}} \Rightarrow \begin{pmatrix} T_L & T_R \\ B_L & B_R \\ t_L & t_R \\ b_L & b_R \\ t'_L & t'_R \end{pmatrix}$$

Parity even : *light chiral fermions*

EW symmetry breaking in SO(5)xU(1)



Predictions

RS : $ds^2 = e^{-2k|y|} dx_\mu dx^\mu + dy^2$

$0 \le |y| \le L = \pi R$

KK mass scale

$$m_{KK} = \frac{\pi k}{e^{kL} - 1} \sim \pi k e^{-kL}$$

W mass

$$m_W \sim \sqrt{\frac{k}{L}} e^{-kL} \sin\theta_H \sim \frac{\sin\theta_H}{\pi\sqrt{kL}} m_{KK}$$

Typically

$$z_L = e^{kL} \sim 10^{15}$$

$$k = 5 \times 10^{17}\,\mathrm{GeV}$$

$$m_{KK} \sim 1.5\,\mathrm{TeV}$$

4D gauge and Higgs couplings

$$F_{MN}{}^2 \sim (\partial_\mu A_\nu - \partial_\nu A_\mu + g[A_\mu, A_\nu])^2$$

WWZ
WWZZ
WWWW

Almost universal in RS

(Large deviation in flat space)

$$+(\partial_\mu A_y - \partial_y A_\mu + g[A_\mu, A_y])^2$$

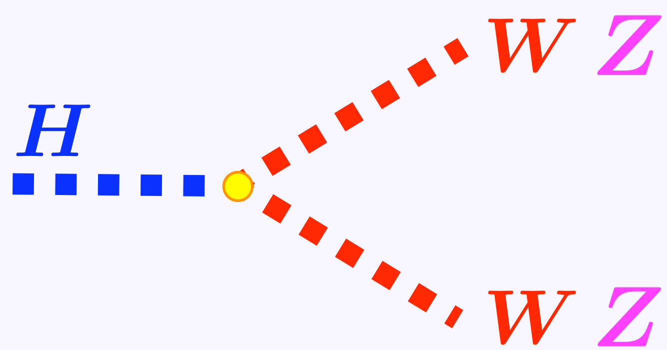
WWH
ZZH
WWHH
ZZHH

significant θ_H -dependence

Sakamura-Hosotani 2006

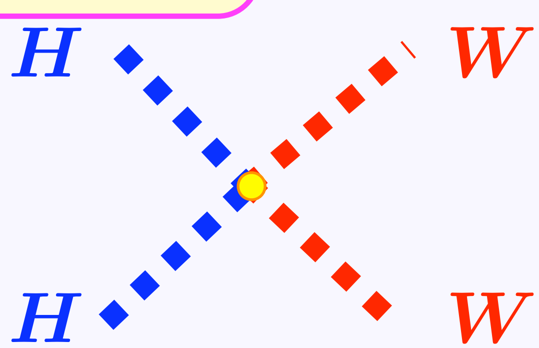
Prediction 1

WWH
ZZH



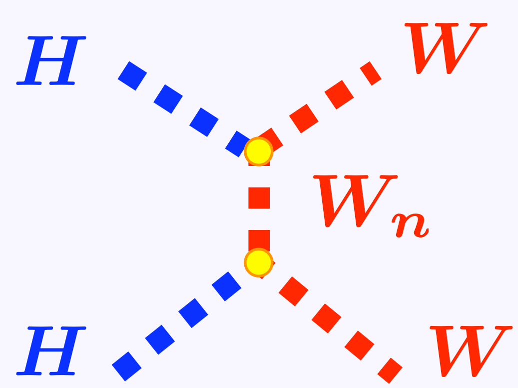
$$SM \times \cos\theta_H$$

WWHH
ZZHH



$$1 - \frac{2}{3}\sin^2\theta_H$$

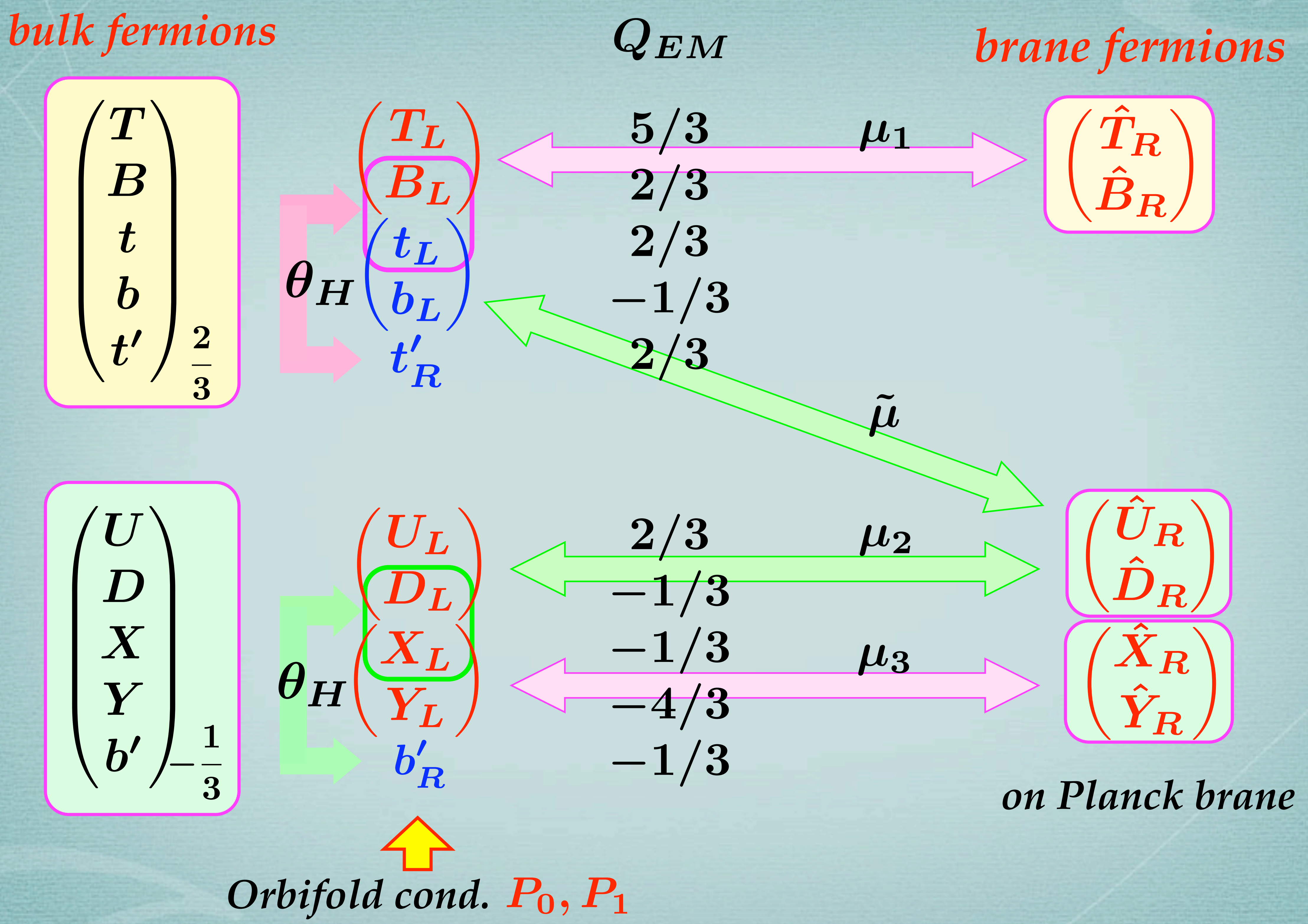
$$+ \sum_n$$



$$-\frac{4}{3}\sin^2\theta_H$$

$$SM \times \cos 2\theta_H$$

Quarks & Leptons

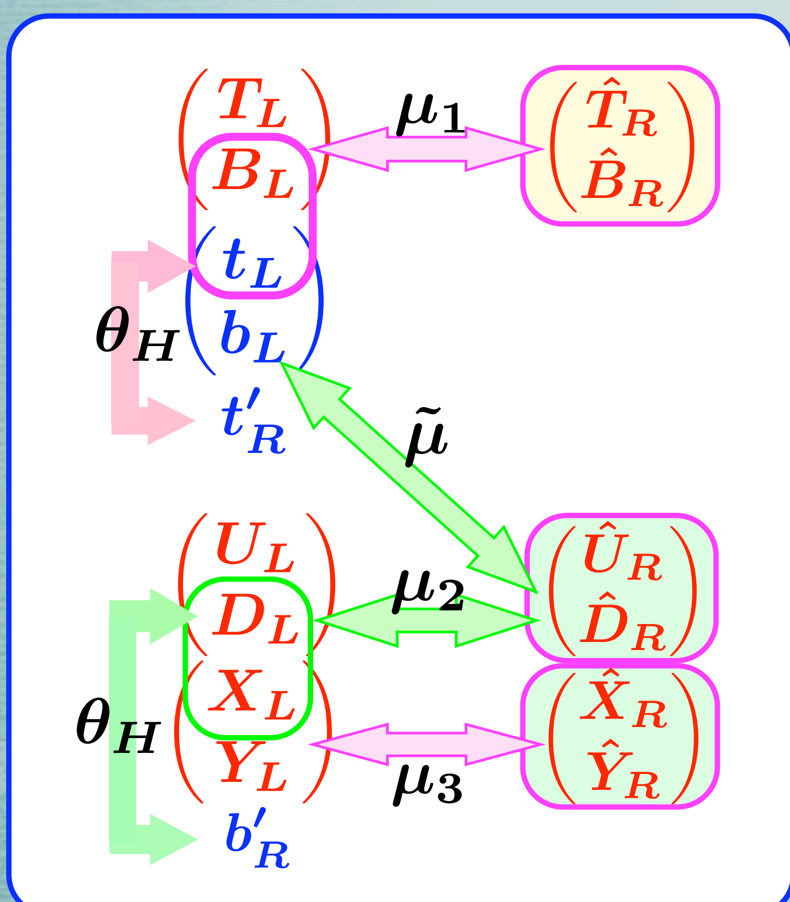

bulk fermions
Q_{EM}
brane fermions
$\begin{pmatrix} T \\ B \\ t \\ b \\ t' \end{pmatrix}_{\frac{2}{3}}$
θ_H
T_L 5/3
B_L 2/3
t_L 2/3
b_L −1/3
t'_R 2/3
μ_1
$\begin{pmatrix} \hat{T}_R \\ \hat{B}_R \end{pmatrix}$
$\tilde{\mu}$
$\begin{pmatrix} U \\ D \\ X \\ Y \\ b' \end{pmatrix}_{-\frac{1}{3}}$
θ_H
U_L 2/3
D_L −1/3
X_L −1/3
Y_L −4/3
b'_R −1/3
μ_2
μ_3
$\begin{pmatrix} \hat{U}_R \\ \hat{D}_R \end{pmatrix}$
$\begin{pmatrix} \hat{X}_R \\ \hat{Y}_R \end{pmatrix}$
on Planck brane
Orbifold cond. P_0, P_1

$$\mathcal{L} = -\overline{\psi}\, i\, \Gamma^a e_a^{\,M} \Big\{ \partial_M + \frac{1}{8}\omega_{bcM}[\Gamma^b, \Gamma^c] - igA_M \Big\} \psi$$

$$- c\, k\, \epsilon(y)\, \overline{\psi}\, \psi \quad \text{+ brane interactions}$$

bulk mass

(Gherghetta-Pomarol 2000)



For $\mu^2 \gg m_{KK} \sim 1.5$ **TeV**

$$m_t \sim \frac{\sqrt{1-4c^2}}{\sqrt{2}\pi}\, |\sin\theta_H| \cdot m_{KK}$$

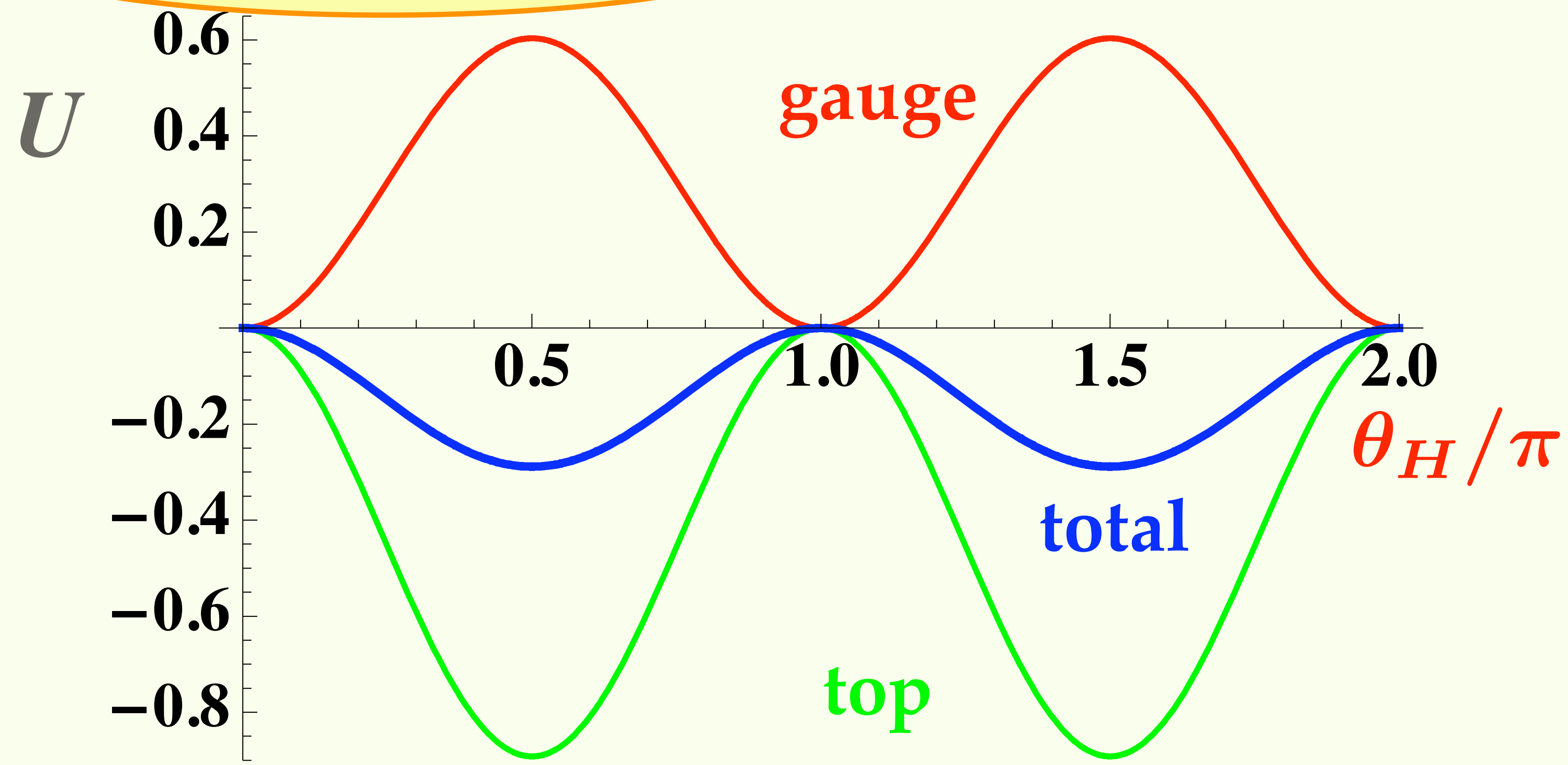
$$c \sim 0.426$$

$$m_b \sim \frac{\tilde{\mu}}{\mu_2} \cdot m_t$$

EW symmetry breaking

Prediction 2

Effective potential



$m_t \sim 172\,\text{GeV}$

$c \sim 0.43$

$V_{\text{eff}}^{\text{gauge}} < -V_{\text{eff}}^{\text{top}}$

Minima at

$\theta_H = \dfrac{\pi}{2}$

Top induces

EW symmetry breaking.

Higgs mass

curvature of $V_{\rm eff}$ at the minimum

$$m_H^2 = \frac{\pi^2 g_4^2 kL}{4 m_{KK}^2} \frac{d^2 V_{\rm eff}}{d\theta_H^2} = \frac{g_4^2 kL}{64\pi^4} \frac{d^2 U}{d\theta_H^2} m_{KK}^2$$

Prediction 3

$z_L = e^{kL}$	k (GeV)	m_{KK} (TeV)	c	m_H (GeV)
10^{17}	5.0×10^{19}	1.58	0.435	54.4
10^{15}	4.7×10^{17}	1.48	0.426	50.8
10^{13}	4.4×10^{15}	1.38	0.413	47.0
10^{10}	3.9×10^{12}	1.21	0.384	40.6
1.30×10^{4}	3.2×10^{6}	0.78	0.	24.5

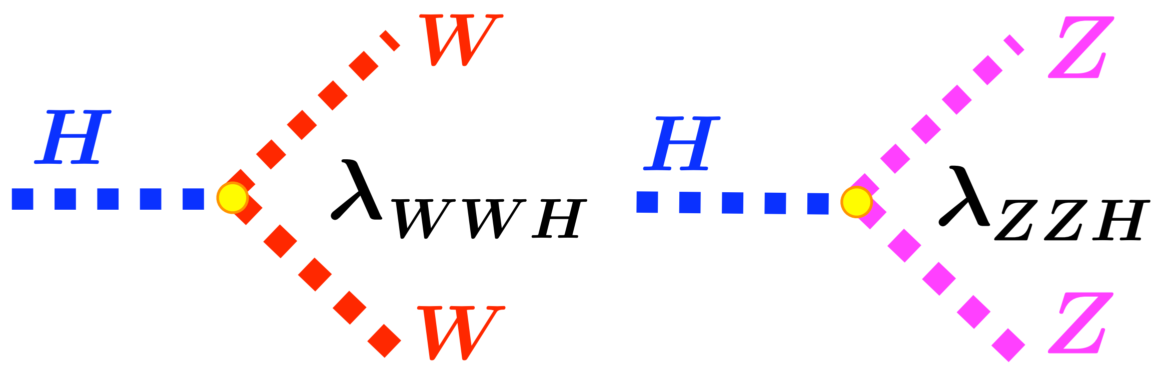
In the minimal model with *brane masses* $\gg m_{KK}$

$m_H \sim 50.8\,\mathrm{GeV}$ **for** $z_L = 10^{15}$

Light Higgs !

But

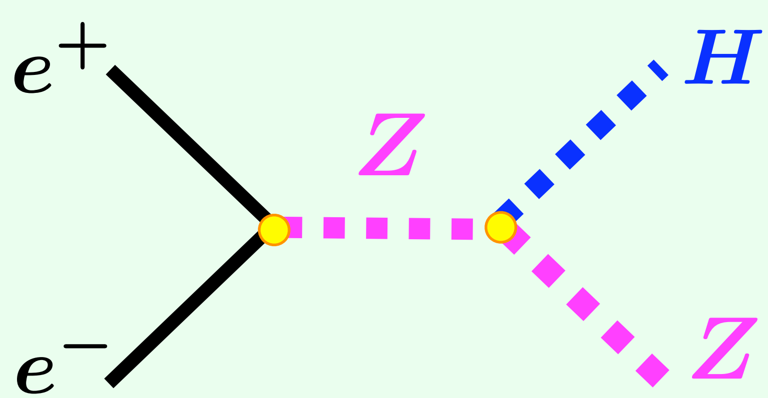
$$\theta_H = \frac{\pi}{2}$$



$$= 0$$

$$\lambda_{ZZH} \simeq \frac{g m_Z}{\cos\theta_W} \cdot \cos\theta_H$$

⇓



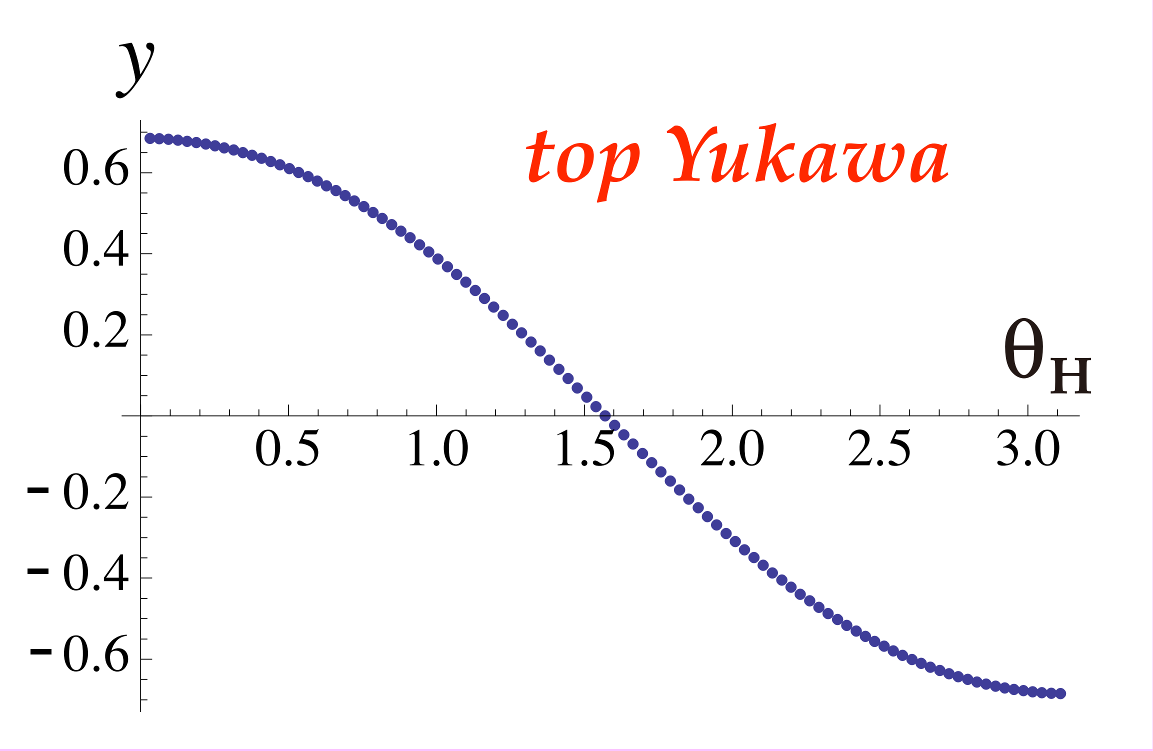
$$= 0$$

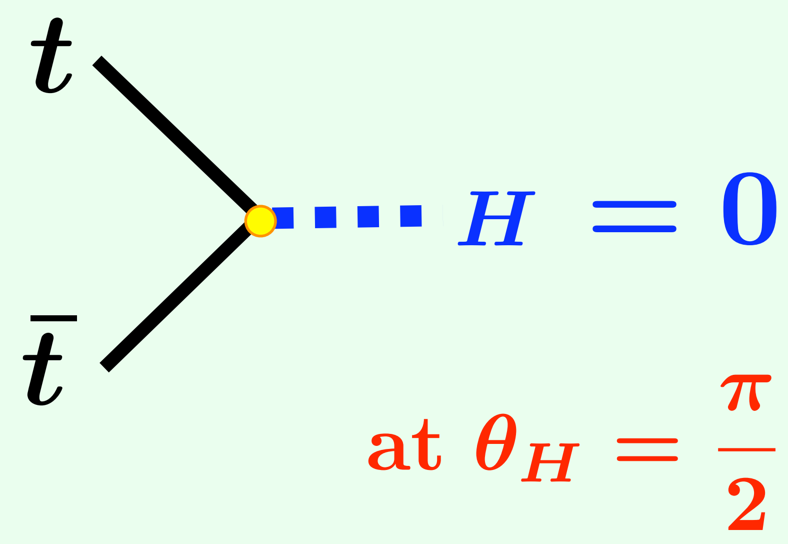
LEP2 bound is evaded.

Similarity to: *Light MSSM Higgs scenario*

Prediction 4

Yukawa couplings *arise from* $g_5\,\overline{\psi}\,\Gamma^5 A_y \psi$





Effective interactions

Effective interactions for

AB phase $\hat{\theta}_H = \theta_H + \frac{H}{f_H}$ $f_H = \frac{2}{\sqrt{kL}} \frac{m_{KK}}{\pi g}$

$\mathcal{L}_{\rm eff} \sim -V_{\rm eff}(\hat{\theta}_H)$ YH 1983

$-m_W(\hat{\theta}_H)^2 W_\mu^\dagger W^\mu - \frac{1}{2} m_Z(\hat{\theta}_H)^2 Z_\mu Z^\mu$

YH-Sakamura 2006, 2007

$-m_f(\hat{\theta}_H)\, \overline{\psi}_f \psi_f$ YH-Kobayashi 2008

$\mathcal{L}_{\rm eff}(\hat{\theta}_H + 2\pi) = \mathcal{L}_{\rm eff}(\hat{\theta}_H)$ *finite*

$$-m_W(\hat\theta_H)^2 W_\mu^\dagger W^\mu - \frac{1}{2} m_Z(\hat\theta_H)^2 Z_\mu Z^\mu$$

SO(5)xU(1) model

$$m_W(\hat\theta_H) \sim \frac{1}{2} g f_H \sin\hat\theta_H$$

$$m_Z(\hat\theta_H) \sim \frac{1}{2\cos\theta_W} g f_H \sin\hat\theta_H$$

$$m_W = \frac{1}{2} g f_H \sin\theta_H \qquad m_Z = \frac{m_W}{\cos\theta_W}$$

WWH	$g_4 m_W \cos\theta_H$	ZZH	$\dfrac{g_4 m_Z}{\cos\theta_W}\cos\theta_H$
$WWHH$	$g_4^2 \cos 2\theta_H$	$ZZHH$	$\dfrac{g_4^2}{\cos^2\theta_W}\cos 2\theta_H$

Deviation from SM

$$-m_f(\hat{\theta}_H)\,\overline{\psi}_f\psi_f$$

HOOS model $\quad m_f(\hat{\theta}_H) \sim \lambda_f \sin\hat{\theta}_H$

$$m_f \sim \lambda_f \sin\theta_H$$

Yukawa $\quad y_f \sim \dfrac{\lambda_f\cos\theta_H}{f_H} = \dfrac{m_f}{2m_W/g}\cos\theta_H$

$$\frac{m_f}{2f_H^2}\,HH\overline{\psi}_f\psi_f$$

Deviation from SM

Light Higgs - $\theta_H = \frac{\pi}{2}$

phenomenology

At $\theta_H = \frac{1}{2}\pi$

Gauge-Higgs couplings

$$\lambda_{WWH} = \lambda_{ZZH} = 0$$

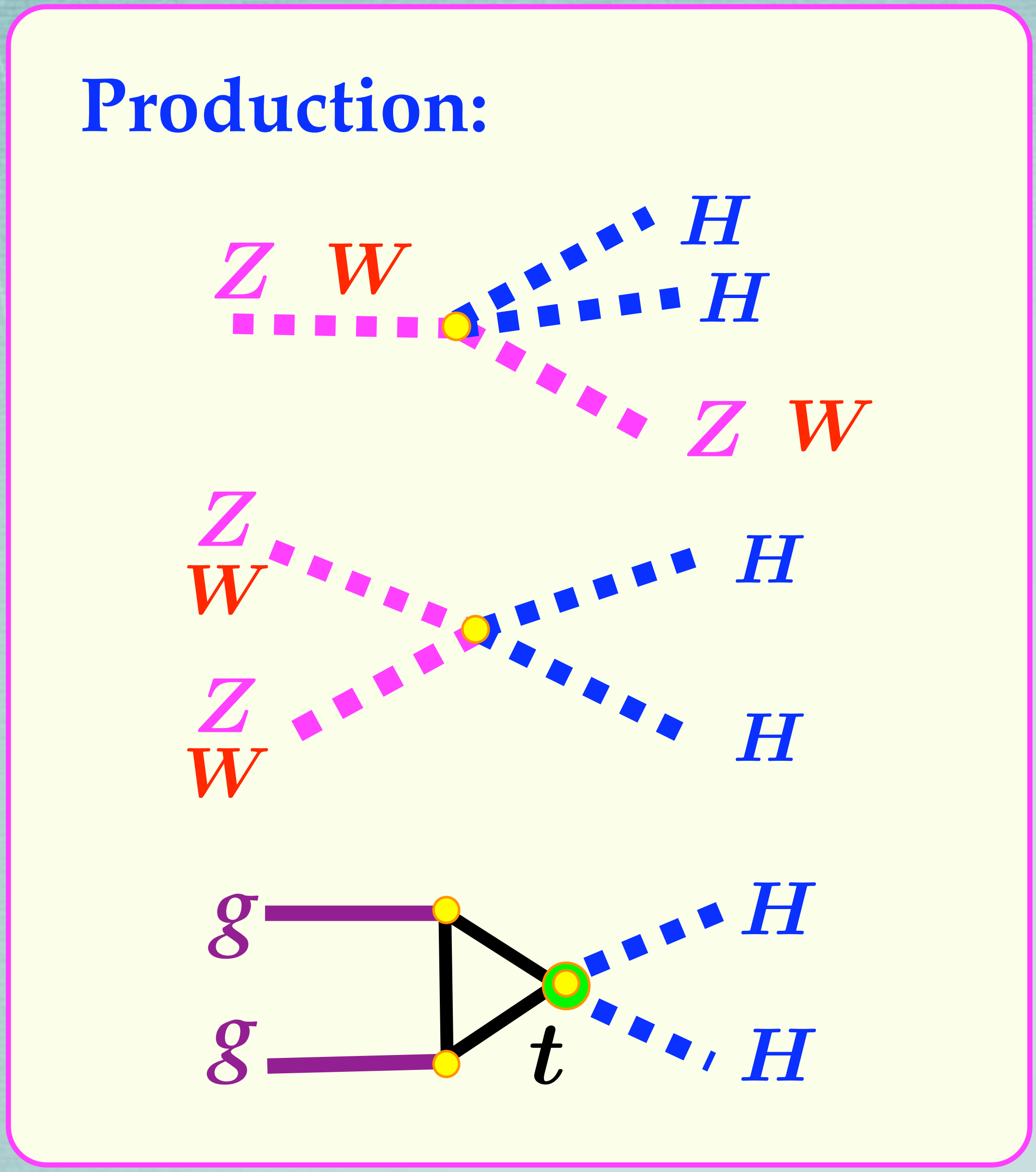
$$\lambda_{WWHH} = -\lambda_{WWHH}^{SM}$$

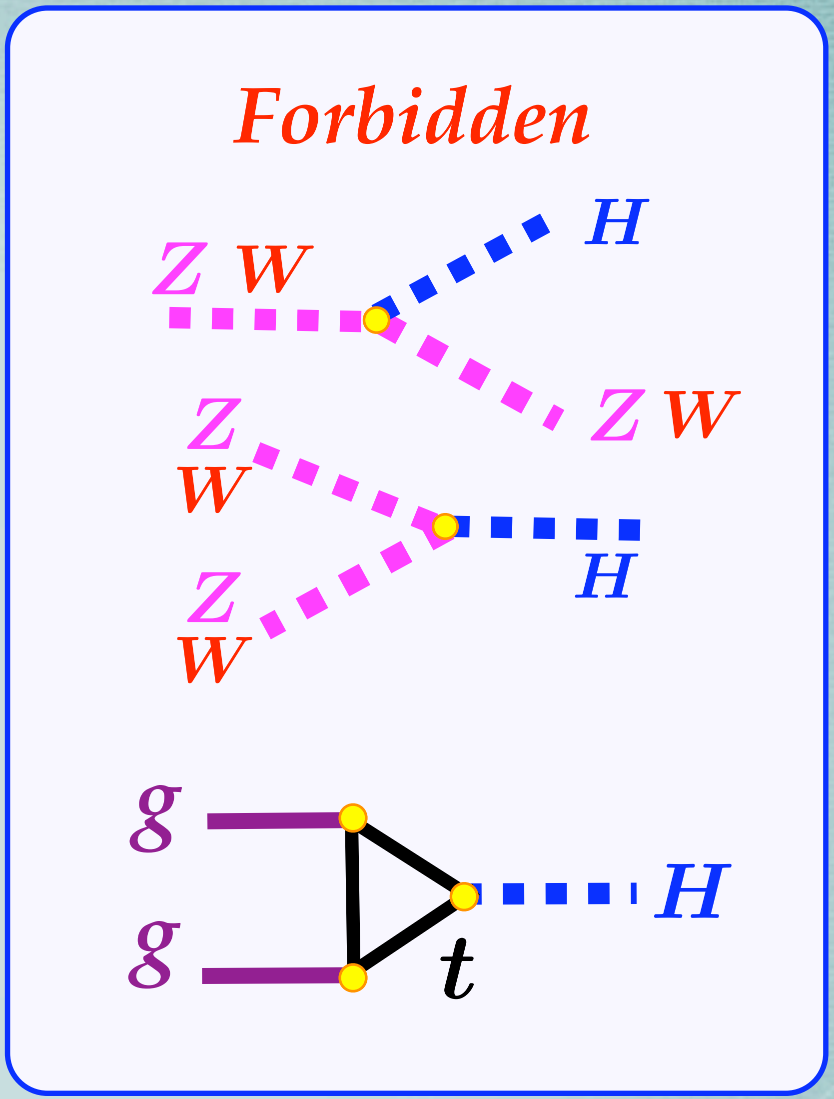
$$\lambda_{ZZHH} = -\lambda_{ZZHH}^{SM}$$

Fermion-Higgs couplings

$$t\bar{t}H \ , \ b\bar{b}H = 0$$

$$t\bar{t}HH \ , \ b\bar{b}HH = \frac{m_t}{f_H^2} \ , \ \frac{m_b}{f_H^2}$$





Higgs particles become stable.

(YH, M.Tanaka)

Summary

Gauge-Higgs Unification

4D Higgs = AB phase in extra dimensions

Top (m_t) ⇨ EW sym breaking $(\theta_H = \frac{1}{2}\pi)$ ⇨ Light Higgs (m_H)

Deviation from SM in the Higgs sector

WWH, ZZH, Yukawa $\propto \cos\theta_H \to 0$

Check at LHC !