

Is lattice simulation of two-dimensional super Yang-Mills possible?

KANAMORI Issaku



Jan. 8, 2009 @ Sapporo Winter School 2009

Refs.

I.K. and H.Suzuki, Nucl.Phys.B in press [arXiv:0809.2856[hep-lat]]

I.K., H.Suzuki and F.Suginio, Phys.Rev. D77 (2008), 091502

I.K., work in progress

Introduction

Lattice Simulation of 2-dim super Yang-Mills is now possible

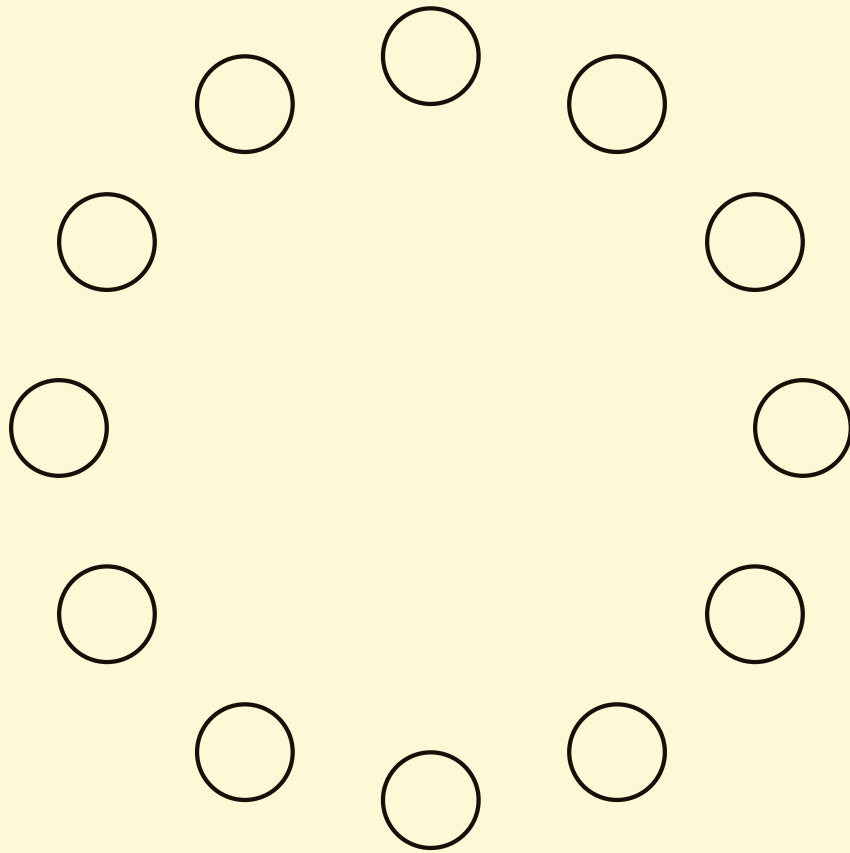
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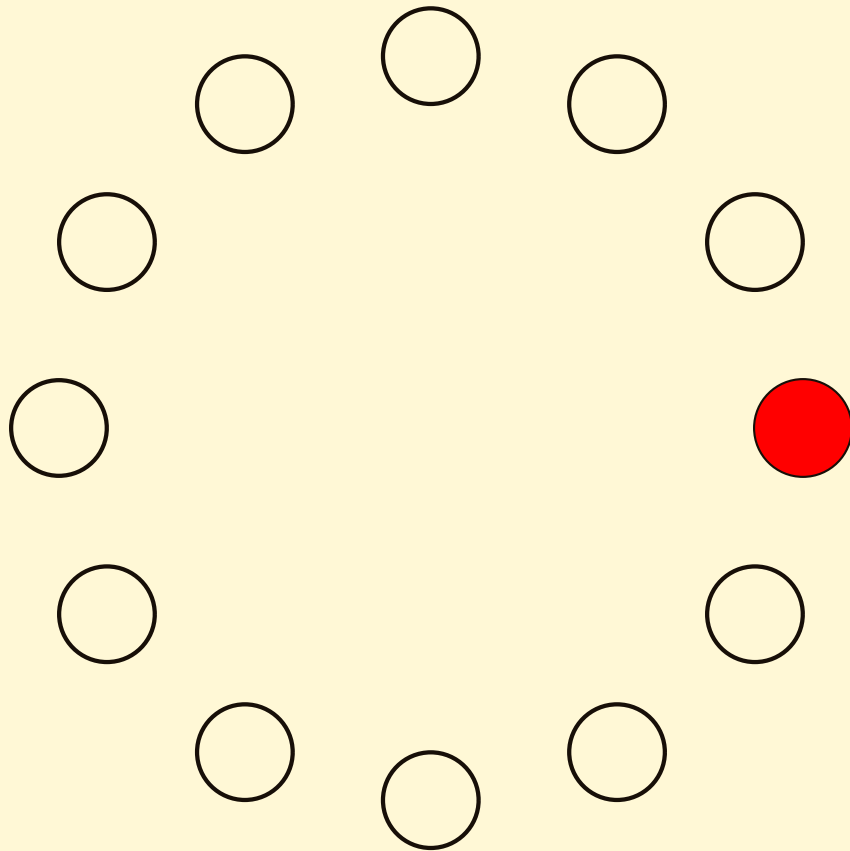
Plan

1. Introduction & Motivation
2. Formulation: Sugino model
3. What should we measure?
 Partially Conserved Super Current (PCSC)
4. Simulation result
5. Application: observing spontaneous SUSY breaking
6. Conclusion

One Example of Spontaneous Symmetry Breaking

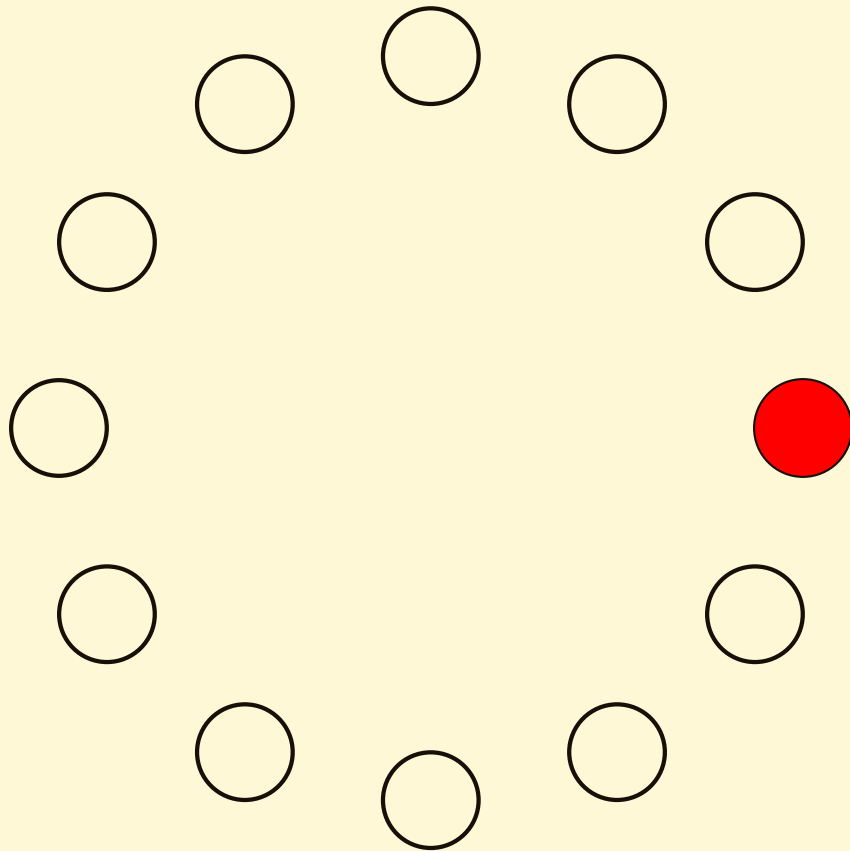


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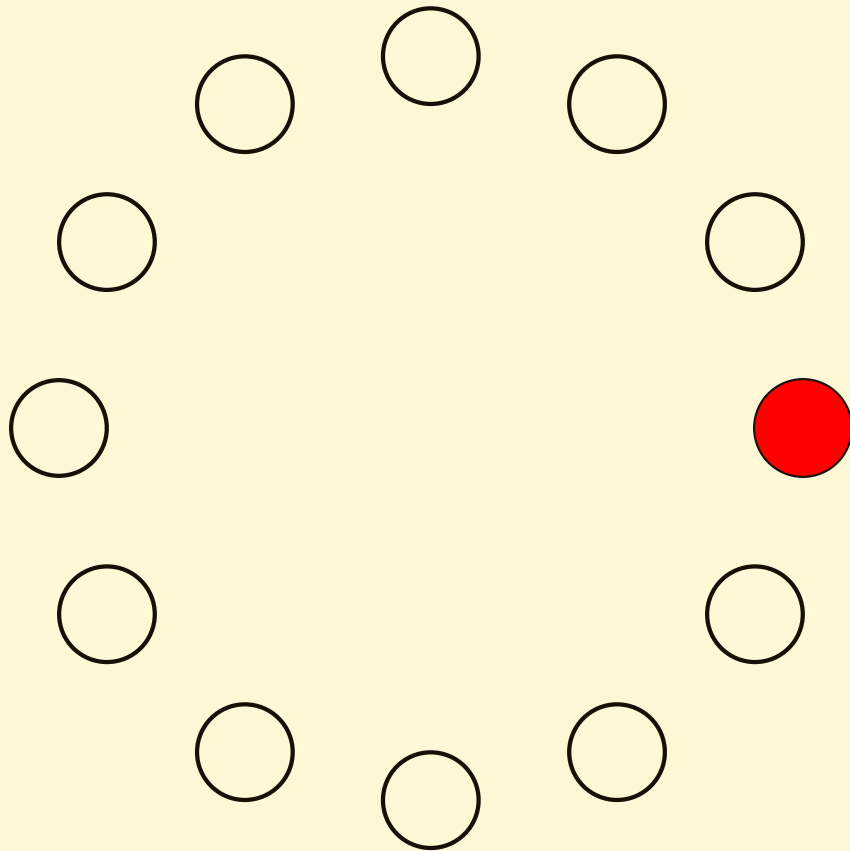
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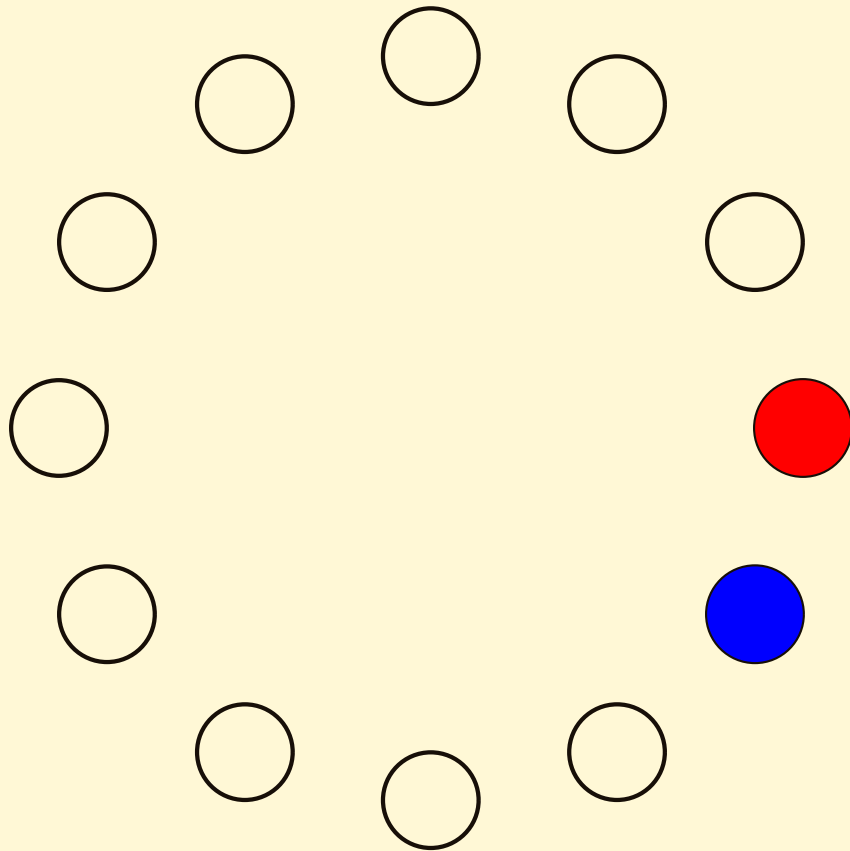
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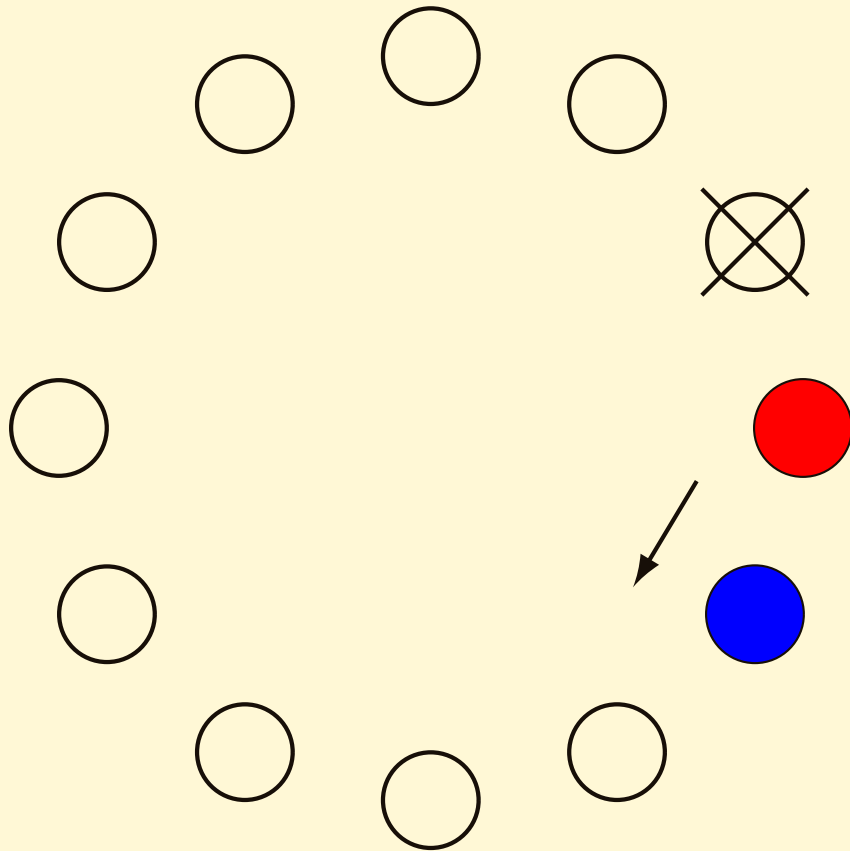
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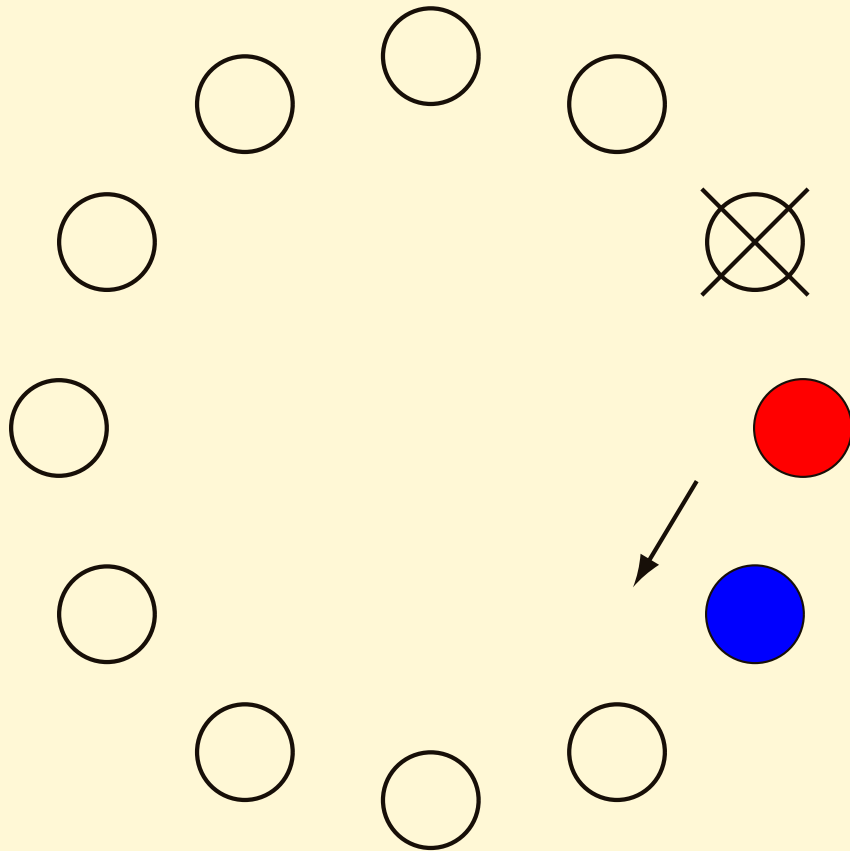
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the poor
first victim

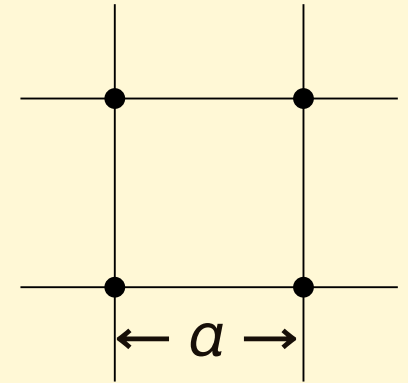
Lattice Scenario à la Kawamoto

Lattice: non-perturbative regularization

continuum limit ($a \rightarrow 0$) in the end

successful in...

- QCD (gauge theory)
- gravity



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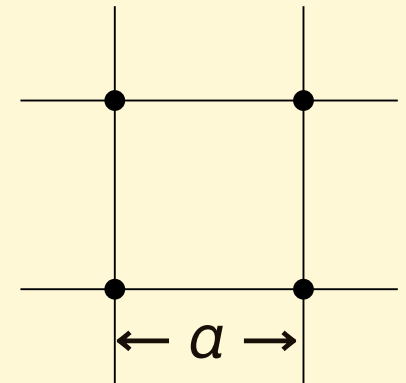
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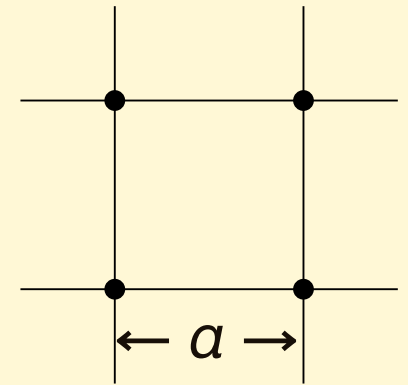
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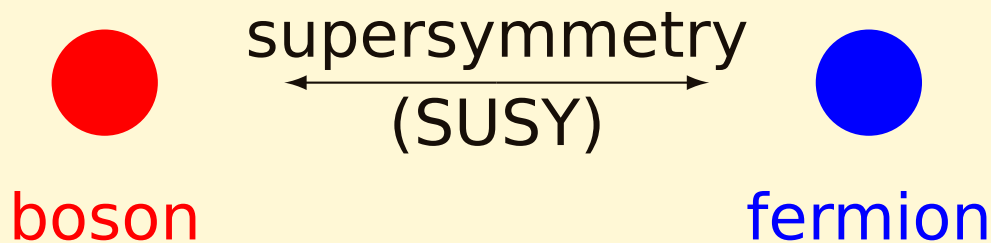
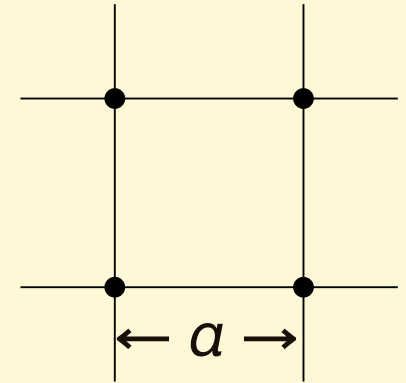
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... treating fermion is not so easy

what is fermions?

\Rightarrow putting supersymmetry = understanding fermions



Motivation

Lattice simulation: a non-perturbative method for field theory

- SUSY breaking
Why our world is *not* supersymmetric?
- gauge/string duality
tool for strong coupling gauge theory
- SUSY can be a symmetry of beyond Standard Model
worth developing simulation techniques
(will be found in LHC ?)
- “Experiment” for theoretical analysis
poster by Suzuki

Formulation

Scalar Q for $\mathcal{N} \geq 2$ on lattice

SUSY on lattice: impossible? ($\because Q \sim \sqrt{P}$)

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If $\mathcal{N} \geq 2$, it is **possible!**

(Models for SYM)

Cohen-Kaplan-Katz-Ünsal, Sugino, Catterall,
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Two types

1. Keep the whole SUSY (at finite a)

DKKN; Nagata; Arianos-D'Adda-Feo-Kawamoto-Saito; Kato-Sakamoto-So

2. partially at finite a , the whole is (automatically)
restored in $a \rightarrow 0$

CKKU; Sugino

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We confirm this scenario in the simulation

Sugino model

Sugino, JHEP 01(2004)067

target: 2-dim $\mathcal{N} = (2, 2)$ SYM

nilpotent Q (Twisted) SUSY Algebra, continuum

$$Q^2 = \delta_{\phi}^{(\text{gauge})} \quad Q_0^2 = \delta_{\bar{\phi}}^{(\text{gauge})} \quad \{Q, Q_0\} = 2i\partial_0 + 2\delta_{A_0}^{(\text{gauge})}$$

Action (dimensional reduction from 4-dim $\mathcal{N} = 1$)

$$S = \frac{1}{g^2} \int d^2x \operatorname{tr} \left\{ \frac{1}{2} F_{MN} F_{MN} + \Psi^T C \Gamma_M D_M \Psi + \hat{H}^2 \right\}$$

$$\Psi^T = (\psi_0, \psi_1, \chi, \eta/2) \quad (\text{under a suitable rep. of } \Gamma_M)$$

$$A_M = (\text{gauge field, scalar})$$

$$\hat{H} = \text{aux. field}$$

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Q -exact action (continuum)

$$S = Q(\dots) = \frac{1}{g^2} \int d^2x \operatorname{tr} \left\{ \frac{1}{4} F_{01}^2 + D_{\mu}\phi D_{\mu}\bar{\phi} + \frac{1}{4} [\phi, \bar{\phi}]^2 \right. \\ \left. + i\psi_{\mu} D_{\mu}\eta + \dots - \frac{1}{4} \eta [\phi, \eta] + \dots \right\}$$

$$\begin{array}{ll} QA_{\mu} = \psi_{\mu} & Q\psi_{\mu} = iD_{\mu}\phi \\ Q\phi = 0 & \dots \end{array} \quad \begin{array}{ll} Q_0A_0 = \frac{i}{2}\eta & Q_0\eta = -2iD_0\bar{\phi} \\ Q_0A_1 = -\chi & Q_0\chi = iD_1\bar{\phi} \\ Q_0\bar{\phi} = 0 & \dots \end{array}$$

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$$QA_{\mu} = \psi_{\mu} \\ Q\phi = 0$$

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$$Q_0\bar{\phi} = 0 \quad \dots$$

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nilpotent Q Lattice version

$$Q^2 = \delta_{\phi}^{(\text{gauge})}$$

Q -exact action (lattice)

$$S = Q(\dots) = S[U(x, \mu), \phi(x), \bar{\phi}(x), H(x), \eta(x), \chi(x), \psi_0(x), \psi_1(x)]$$

bosons
fermions

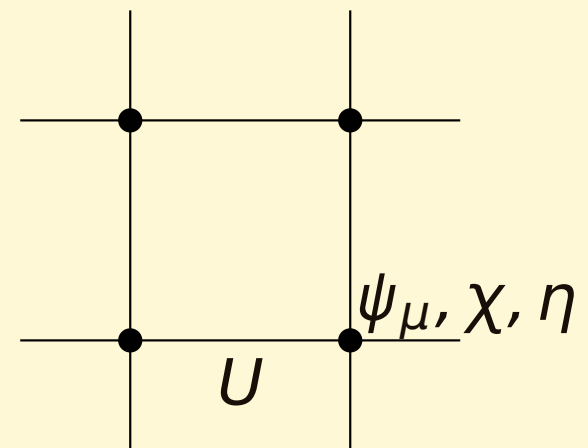
$$QU(x, \mu) = i\psi_{\mu}(x)U(x, \mu)$$

$$Q\psi_{\mu}(x) = i\psi_{\mu}(x)\psi_{\mu}(x)$$

$$-i(\phi(x) - U(x, \mu)\phi(x + \hat{\mu})U(x, \mu)^{-1})$$

$$Q\phi = 0$$

\vdots



Symmetries

(continuum) \Rightarrow (lattice)

- 4 supercharges: $\{Q_0, Q_1, \tilde{Q}, Q\} \Rightarrow Q$
- R-symmetry: $U(1)_A, U(1)_V, Z_2 \Rightarrow U(1)_A$

($U(1)_A$: a rotational symmetry on 2-3 plane)

Scenario

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We will confirm this scenario

What should we measure?
to check the restoration of SUSY

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- perturbative power counting: non-pertubativity...?

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Powerful non-perturbative tool

- Simulation! : conservation of “super current”

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Target: 2-dim $\mathcal{N} = (2, 2)$ SYM, SU(2)
lattice model + scalar mass term
+ thermal B.C.

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PCSC relation

(Separate the effect of lattice artifact)

satisfied \Rightarrow the lattice artifact vanishes

not satisfied \Rightarrow does not vanish

PCSC relation

4 supercharges: $Q_A = \{Q_0, Q_1, \tilde{Q}, Q\}$

Partially conserved supercurrent:

$$\partial_\mu \mathcal{J}_\mu^A = 0 \Rightarrow \boxed{\partial_\mu \mathcal{J}_\mu^A = \mu^2/g^2 Y^A} \text{ (PCSC) } \quad \mu: \text{ scalar mass}$$

$$\langle \partial_\mu \mathcal{J}_\mu^A(x) X^A(0) \rangle - \frac{\mu^2}{g^2} \langle Y^A(x) X^A(0) \rangle = -i\delta^2(x) \langle Q^A X^A(0) \rangle$$

$$\text{measure } \frac{\langle \partial_\mu \mathcal{J}_\mu^A(x) X^A(0) \rangle}{\langle Y^A(x) X^A(0) \rangle} \text{ as a function of } \mu^2/g^2$$

$$Y^A = -2[C(\Gamma_2 \text{tr}(A_2 \Psi) + \Gamma_3 \text{tr}(A_3 \Psi))]^A \\ \sim (\text{scalar}) \times (\text{fermion})$$

$$X^A = \frac{1}{g^2} [\Gamma_0(\Gamma_2 \text{tr}(A_2 \Psi) + \Gamma_3 \text{tr}(A_3 \Psi))]^A$$

Possible Divergences: No Problem

IF the 2 point funcs. have divergence...

$\mathcal{O}(a)$ lattice artifacts in \mathcal{J} and Y

$\Rightarrow \mathcal{O}(a) \times \infty$ can be finite

\Rightarrow need to remove such extra effects

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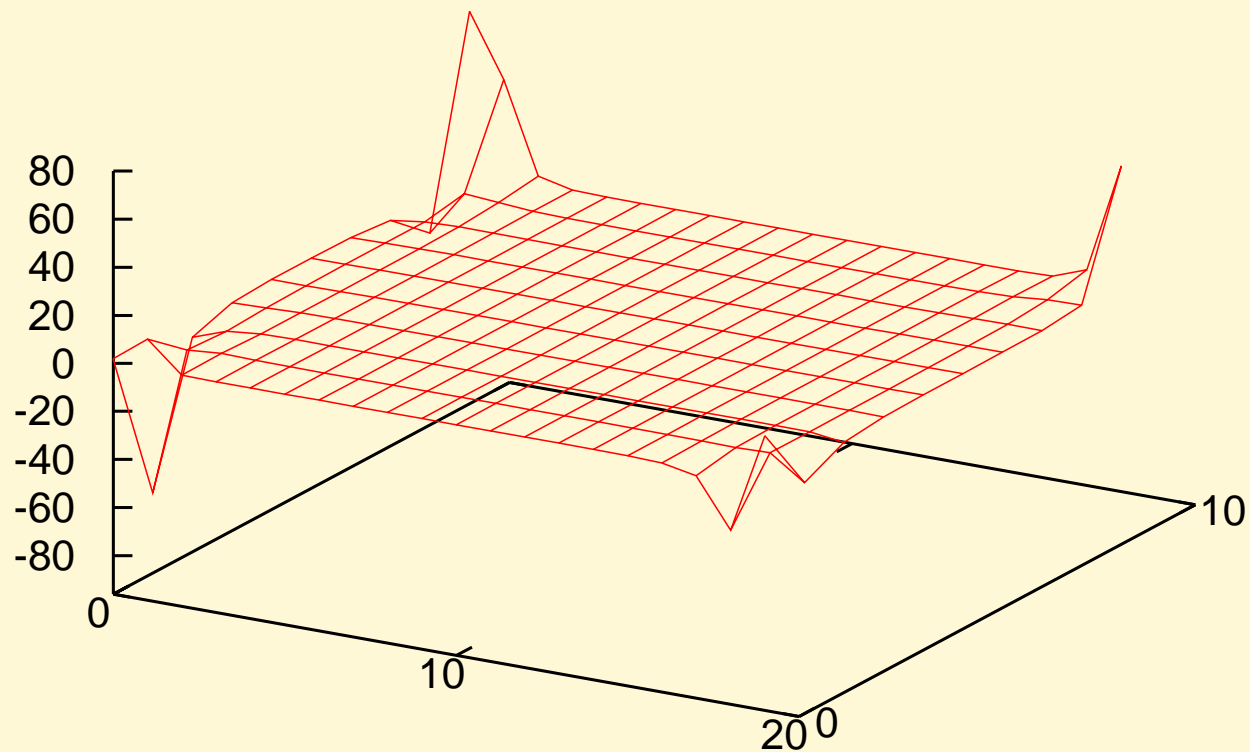
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Simple analysis is OK

Simulation Result

PCSC relation

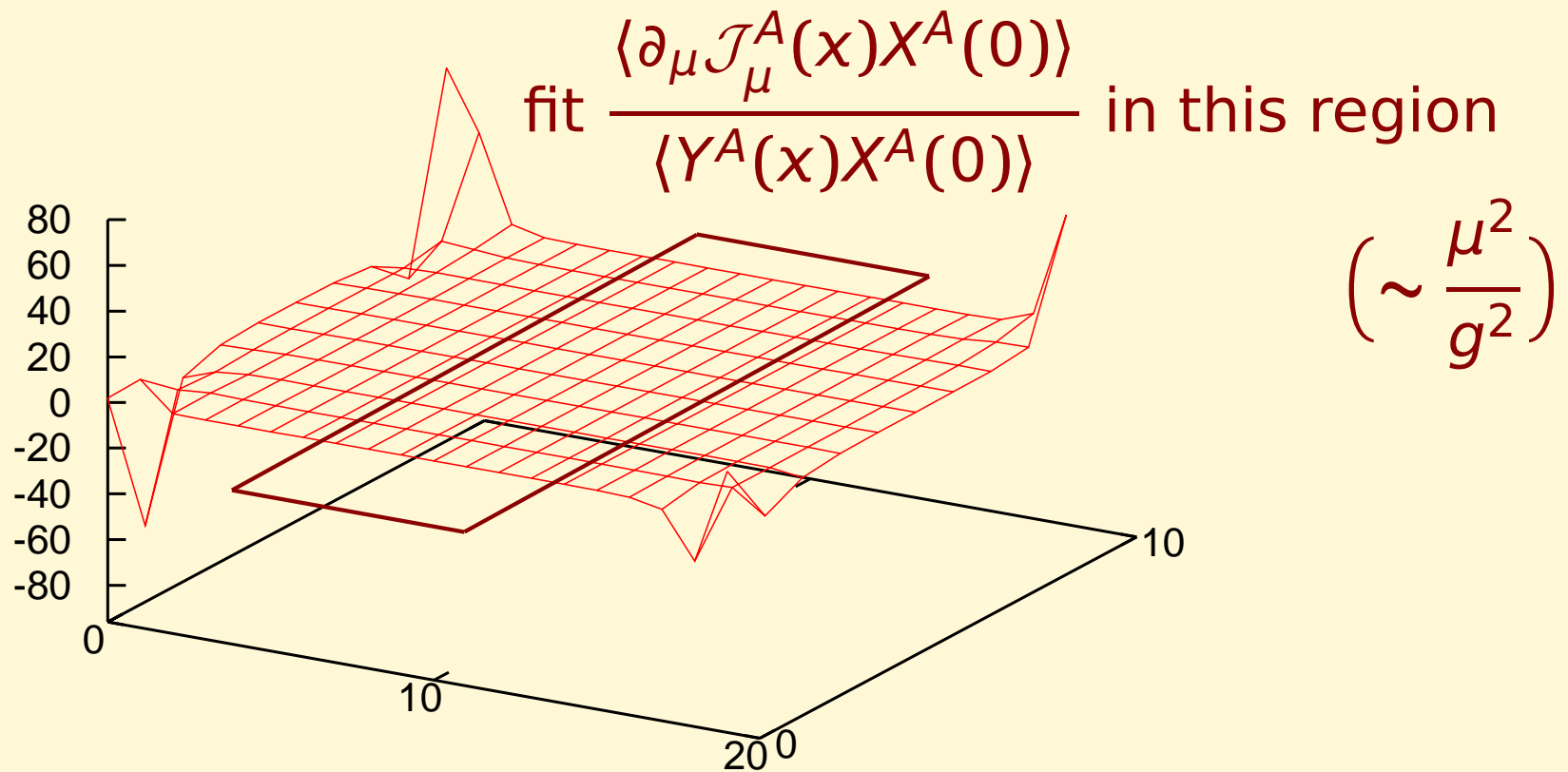
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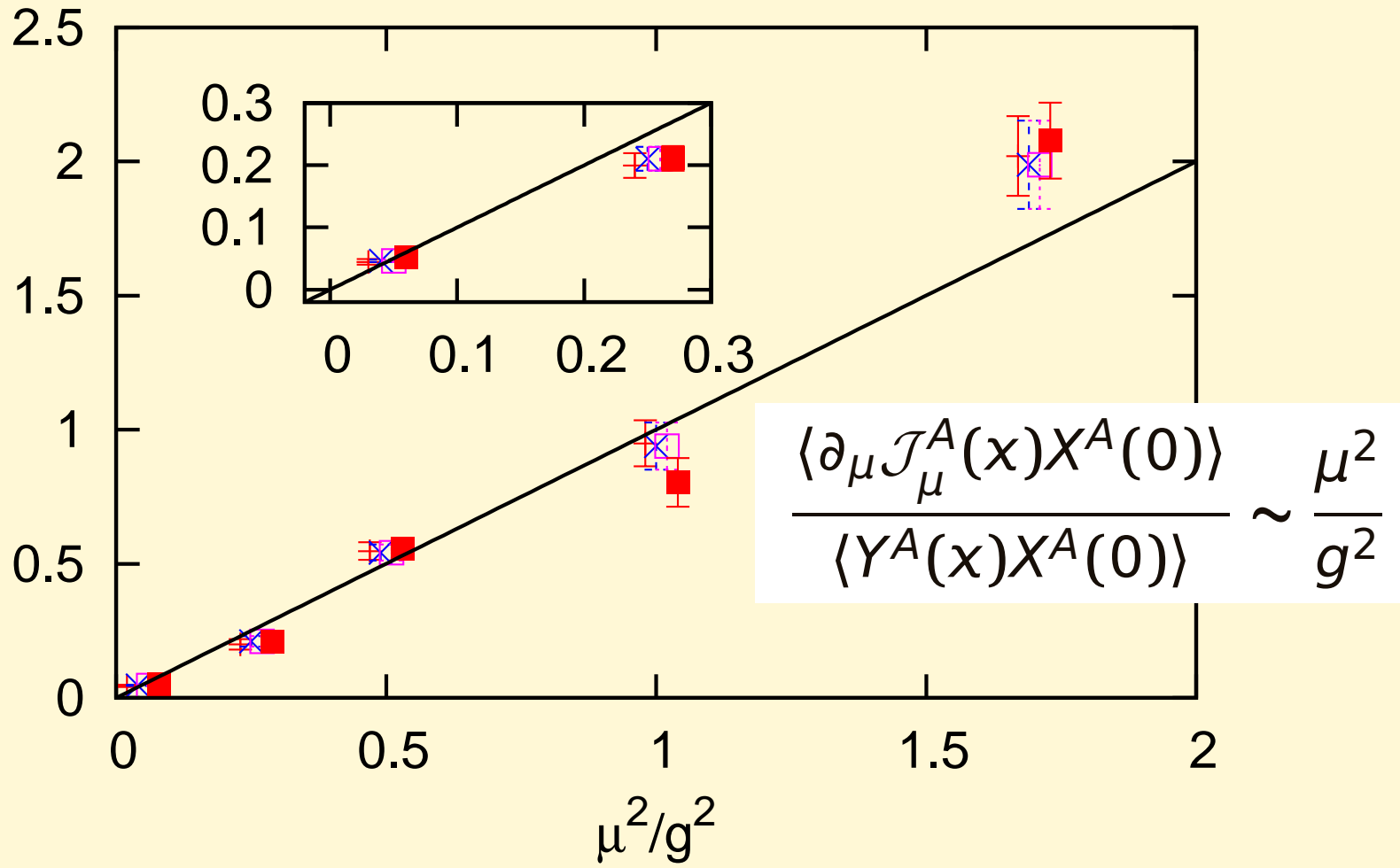
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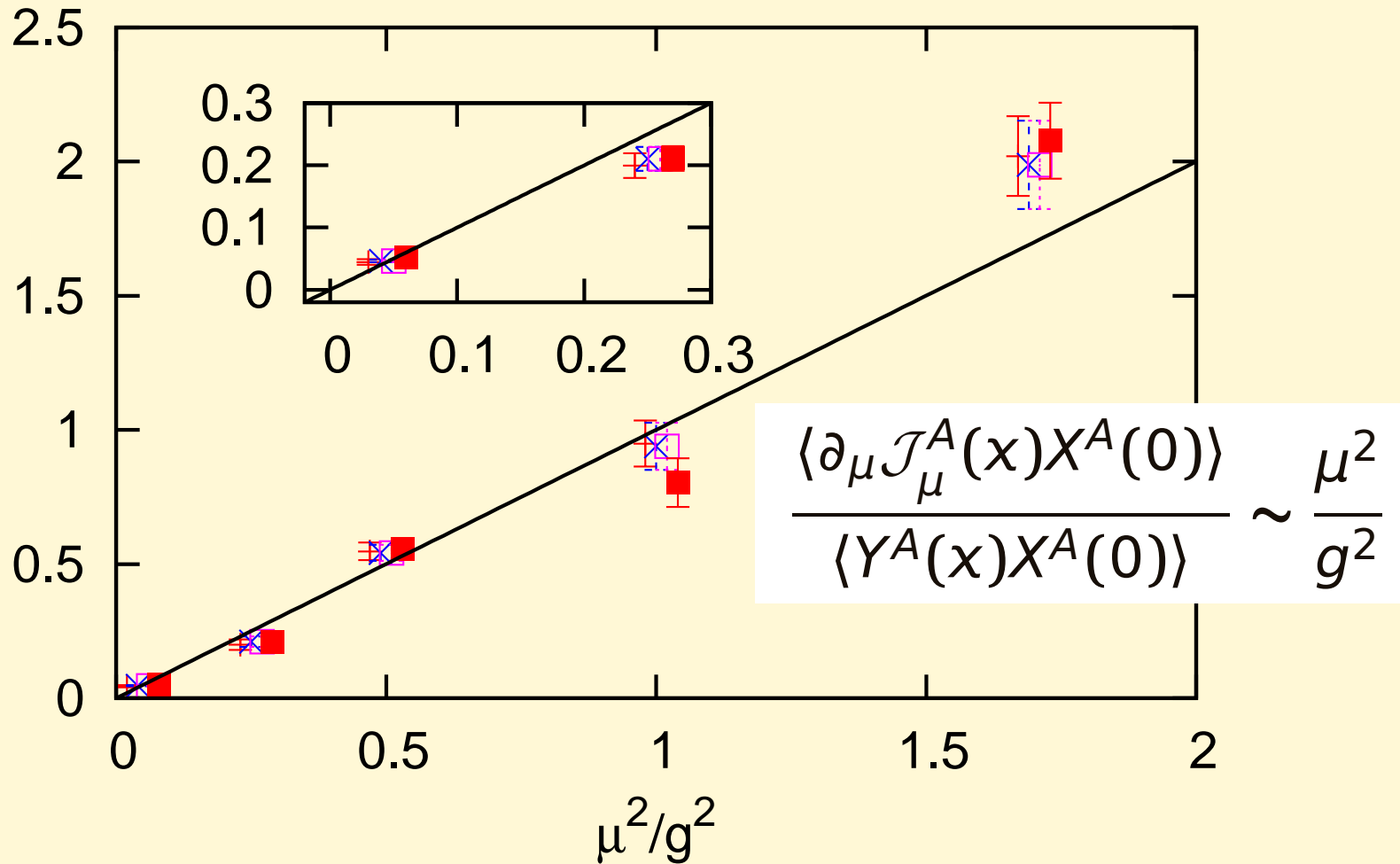


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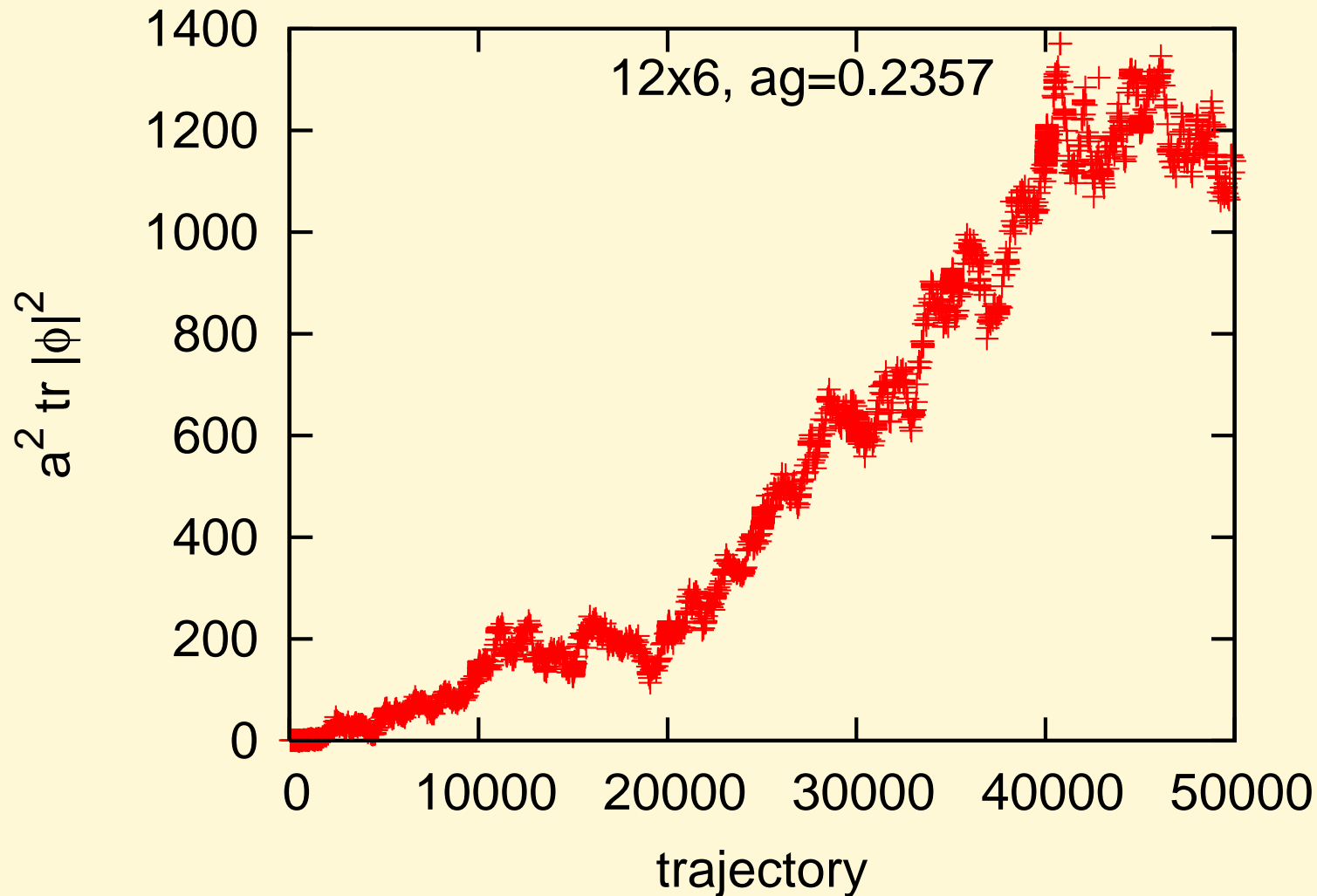
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PCSC is satisfied \Rightarrow no SUSY breaking due to lattice artifact

massless case: never thermalized

because of the flat direction in the potential

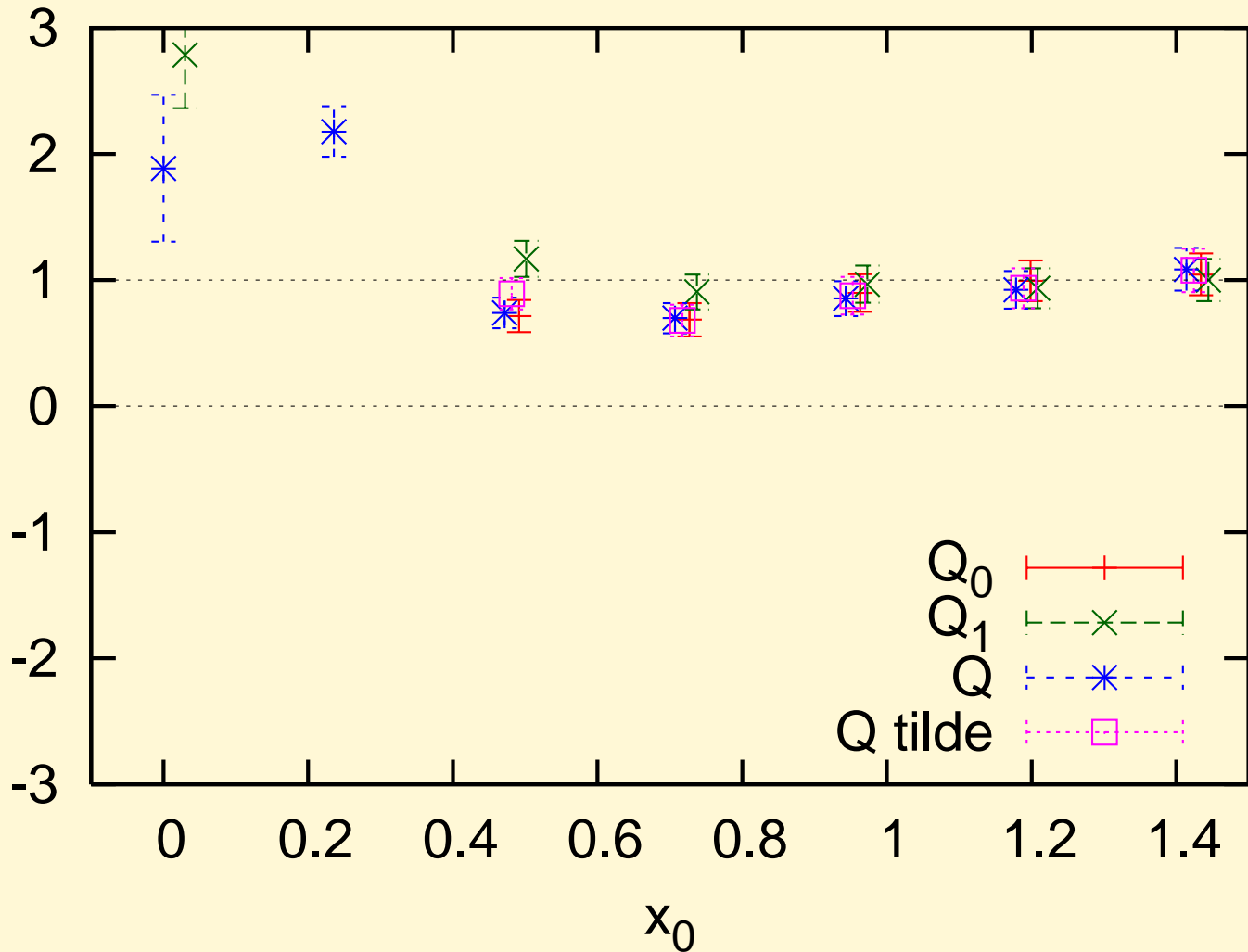
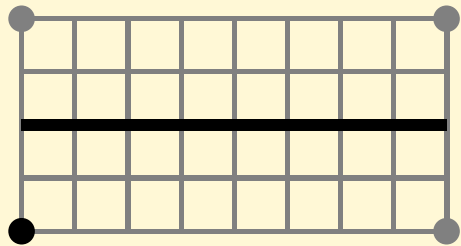


cannot be used in the measurement

anti-PBC \Rightarrow PBC: Noisy

$$\frac{\langle \partial_\mu \mathcal{J}_\mu^A(x) X^A(0) \rangle}{\langle Y^A(x) X^A(0) \rangle} \sim \frac{\mu^2}{g^2} + \delta(x) \quad \mu^2/g^2 = 1, \text{ anti-PBC}$$

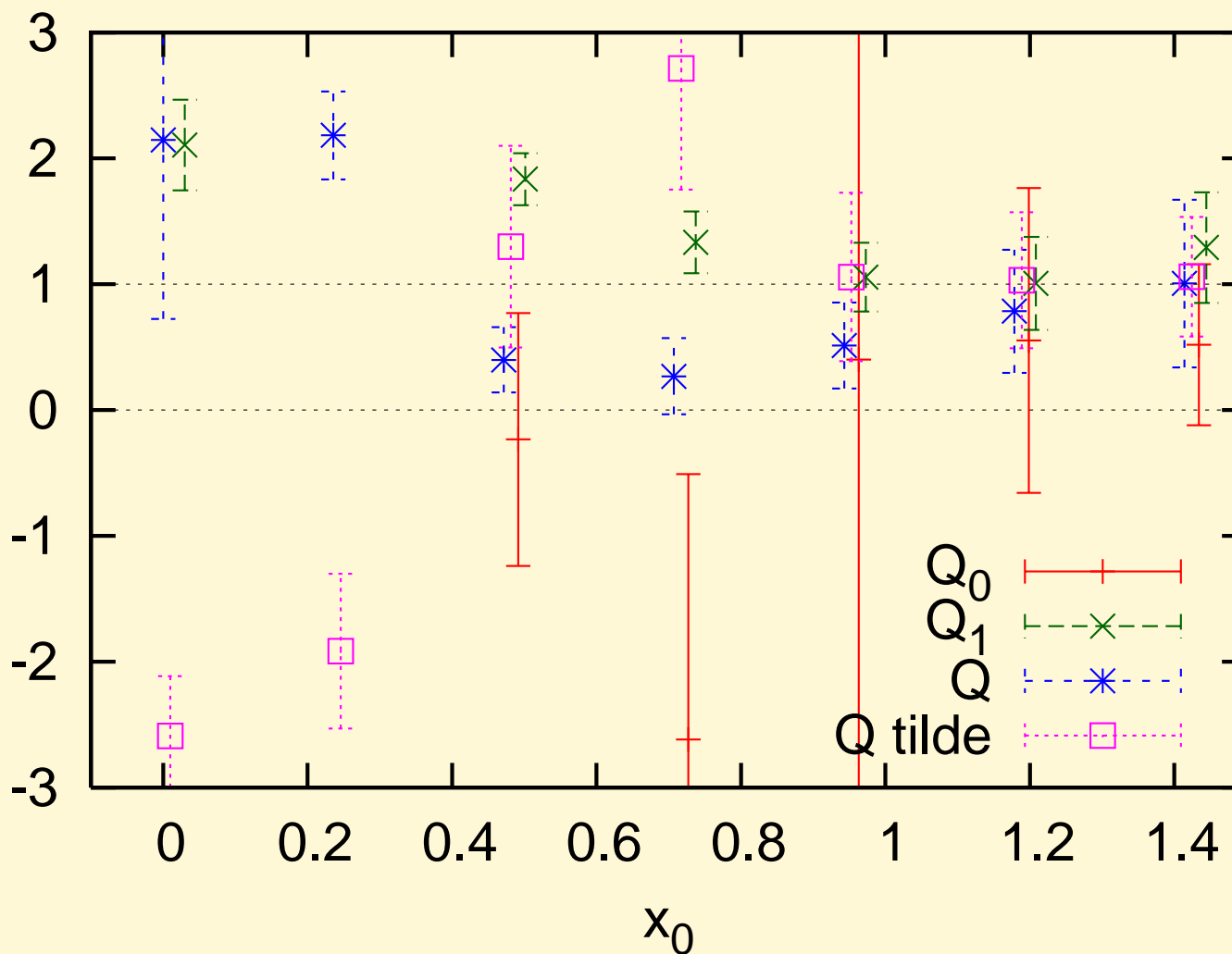
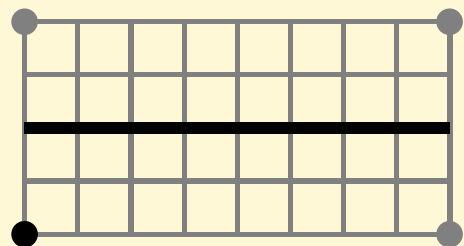
12x6
 $ag = 0.2367$



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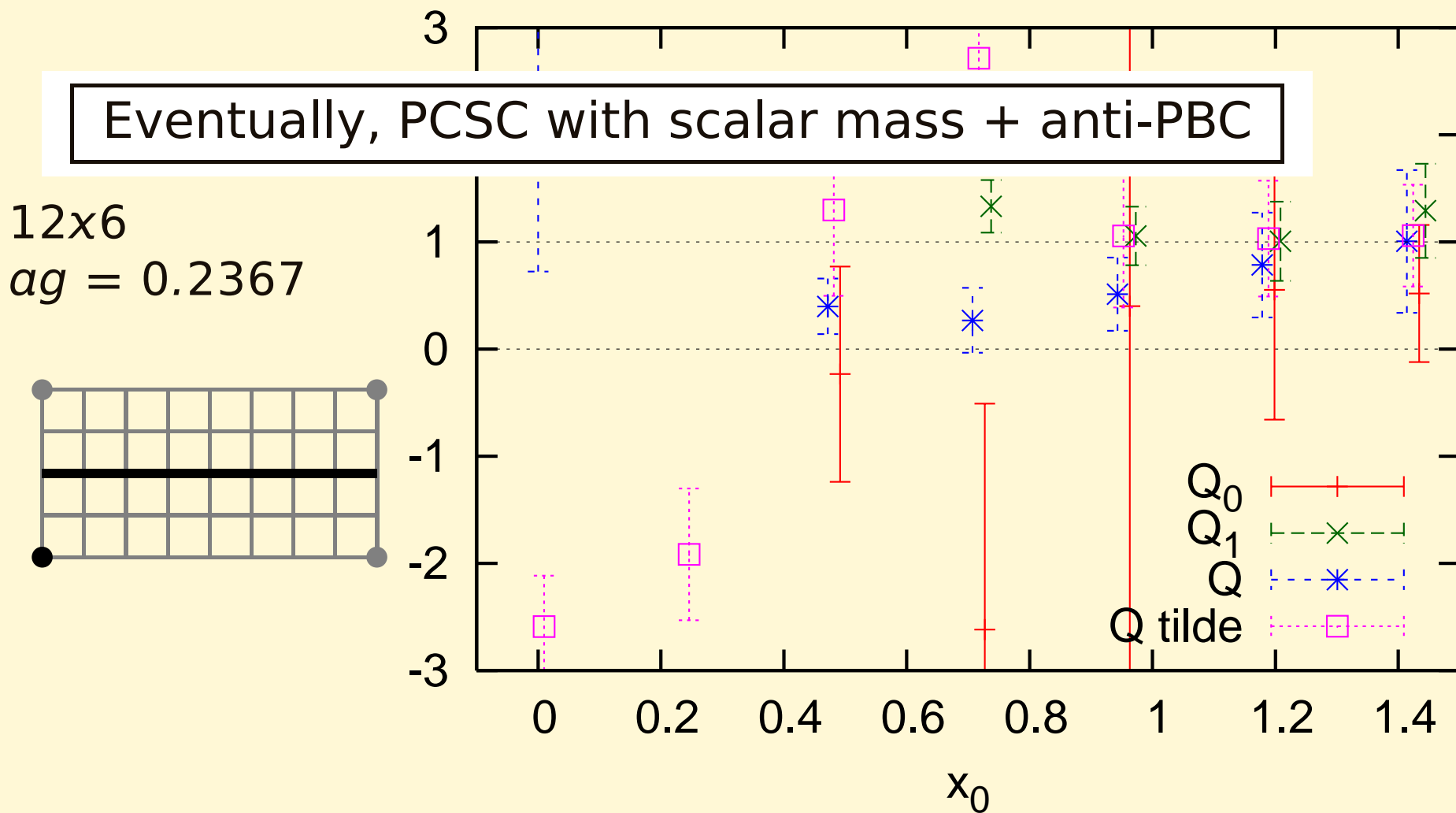
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Simulation detail

- Computer: Riken Super Combined Cluster (RSCC)
- Algorithm: Rational Hybrid Monte Carlo (RHMC)
+ Multi-time step acceleration
- lattice size: $12 \times 6-30 \times 10$
- $ag = 0.2357, 0.2, 0.1768, 0.1414$
- 800–1,800 configurations

Applications

Choice of the scalar mass

PCSC relation \Rightarrow lattice simulation works

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- Observing spontaneous SUSY breaking

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... before that,

flat direction? (How to “define” the theory)

- Define as an extrapolation of $\mu \rightarrow 0$
- Small mass: $\mu < 1/L$
- large N_C
- (unstable)

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Observing SUSY breaking

cf. I.K.-Suzuki-Sugino

- order parameter $\langle \mathcal{H} \rangle = \frac{1}{2} \langle Q \mathcal{J}_0^{(0)} \rangle$ ($\{Q, Q_0\} = 2i\partial_0$)
- measure under thermal boundary condition
- extrapolate the scalar mass to zero
- extrapolate to zero temperature:
ground state energy density \mathcal{E}

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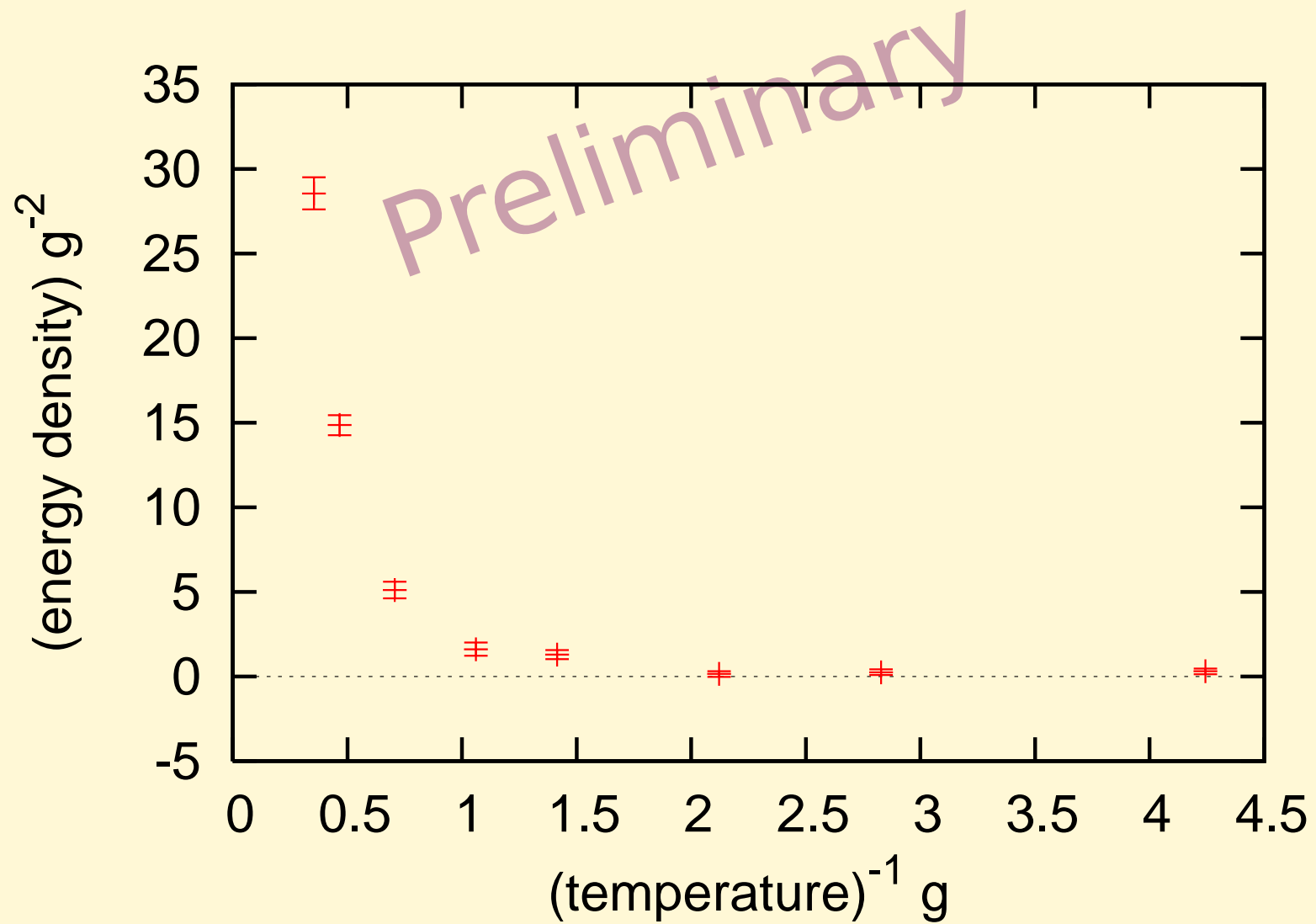
cf. I.K.-Suzuki-Sugino

- order parameter $\langle \mathcal{H} \rangle = \frac{1}{2} \langle Q \mathcal{J}_0^{(0)} \rangle$ ($\{Q, Q_0\} = 2i\partial_0$)
- measure under thermal boundary condition
- extrapolate the scalar mass to zero
- extrapolate to zero temperature:
ground state energy density \mathcal{E}

$$\mathcal{E} = \langle \mathcal{H} \rangle \text{ at zero temperature } \begin{cases} = 0 & \text{SUSY} \\ \neq 0 & \text{SUSY} \end{cases}$$

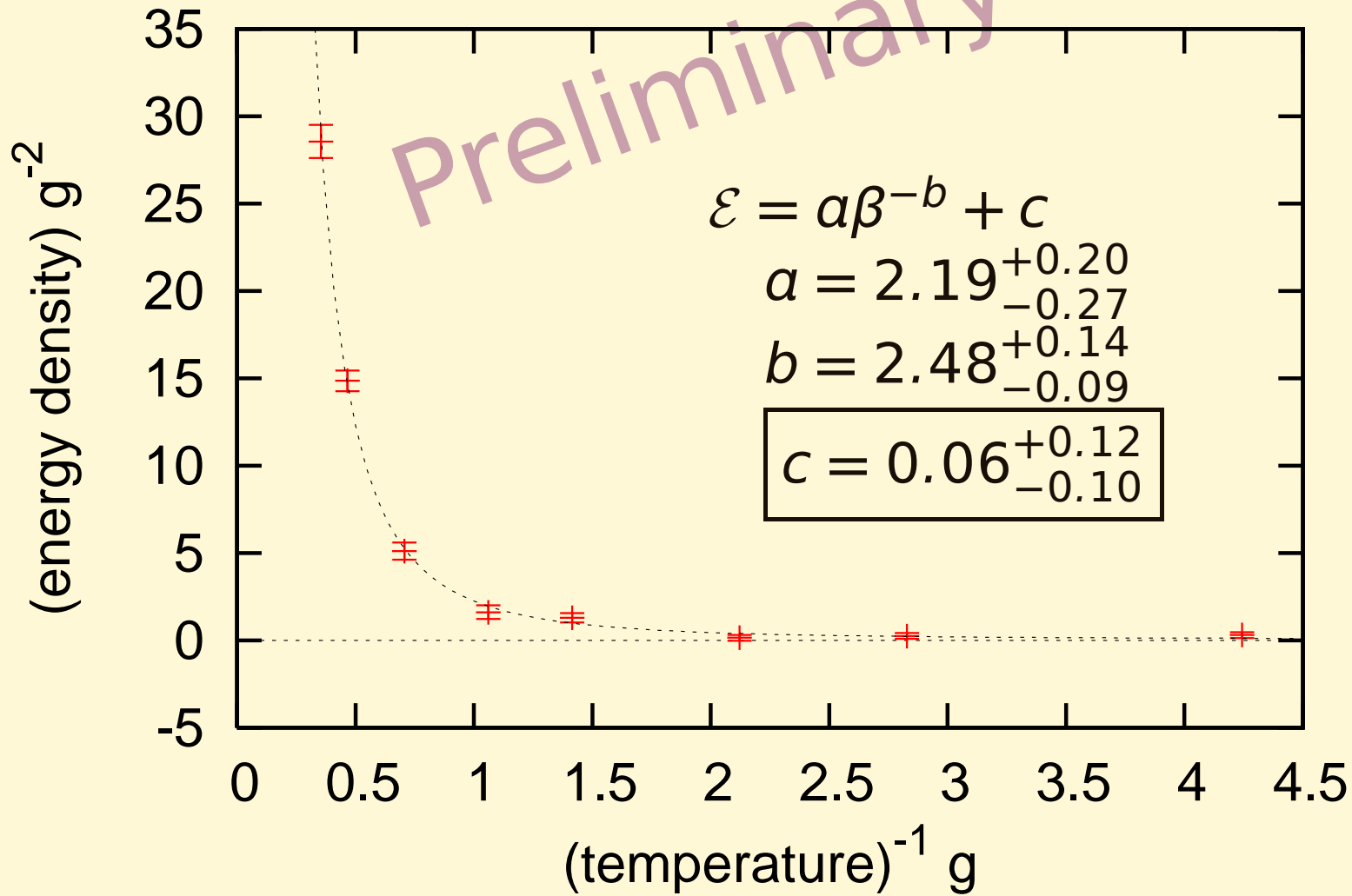
Cf. No analytic result for this system
(maybe broken? a conjecture by Hori-Tong)

not broken?



not broken?

Ground state energy is consistent with 0



Conclusion

Conclusion

Lattice simulation of 2-dim super Yang-Mills is now possible

Formulation with 1 nilpotent Q

\Rightarrow whole SUSY is restored in the cont. limit (no lat. artifact)

- simulation: 2-dim $\mathcal{N} = (2, 2)$ SYM, $SU(2)$ (Sugino model)
- scalar mass term \Rightarrow Partially Conserved Super Current (PCSC) relation

Application

- Observing (no) spontaneous SUSY breaking
- \vdots (Poster by Suzuki)

Thank you.