Hadron Structure from Lattice QCD

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– QCDSF Collaboration –

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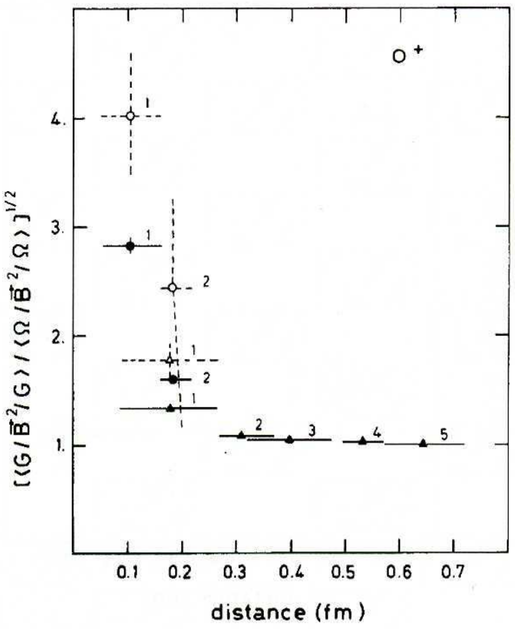


Fig. 4. Cross section of the 0^+ glueball. The closed (open) triangles correspond to $\beta = 2.3$, $\Delta t = 0$ ($\beta = 2.3$, $\Delta t = a$) and the closed (open) circles correspond to $\beta = 2.5$, $\Delta t = 0$ ($\beta = 2.5$, $\Delta t =$ a). The location of the corresponding plaquettes within the glueball is indicated by the numbers; see fig. 3.

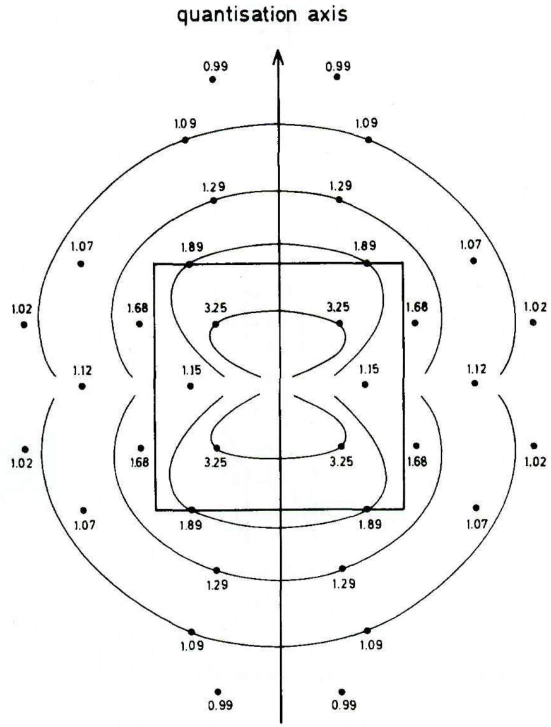


Fig. 7. Equi-potential contour plot of the colour magnetic field inside the 2^+ glueball. The dots correspond to the location of the various plaquettes (figs. 3b and c), rotated into one plane. The numbers give the ratios $[\langle 2^+|\boldsymbol{B}^2|2^+\rangle / \langle\Omega|\boldsymbol{B}^2|\Omega\rangle]^{1/2}$ for $\beta = 2.5$ and $\Delta t = 0$. Also drawn is the 2×2 plaquette which sets the scale: $2a \approx 0.3$ fm.

8^4 Lattice $SU(2)$ (quenched)

Outline

Lattice Simulation

Basics

Nucleon

Pion

Conclusions & Outlook

Lattice Simulation

Action

$N_f = 2$

$$S = S_G + S_F$$

$$S_G = \beta \sum_{x,\mu<\nu} \Big(1 - \frac{1}{3}\mathrm{Re\,Tr}\, U_{\mu\nu}(x)\Big)$$

$$S_F = \sum_x \Big\{ \bar\psi(x)\psi(x) - \kappa\, \bar\psi(x) U_\mu^\dagger(x-\hat\mu)[1+\gamma_\mu]\psi(x-\hat\mu) - \kappa\, \bar\psi(x) U_\mu(x)[1-\gamma_\mu]\psi(x+\hat\mu) - \frac{1}{2}\kappa\, c_{SW}\, g\, \bar\psi(x)\sigma_{\mu\nu}F_{\mu\nu}(x)\psi(x) \Big\}$$

$\Updownarrow$

$\partial_\mu A_\mu^{\mathrm{imp}} = 2 m_q P$

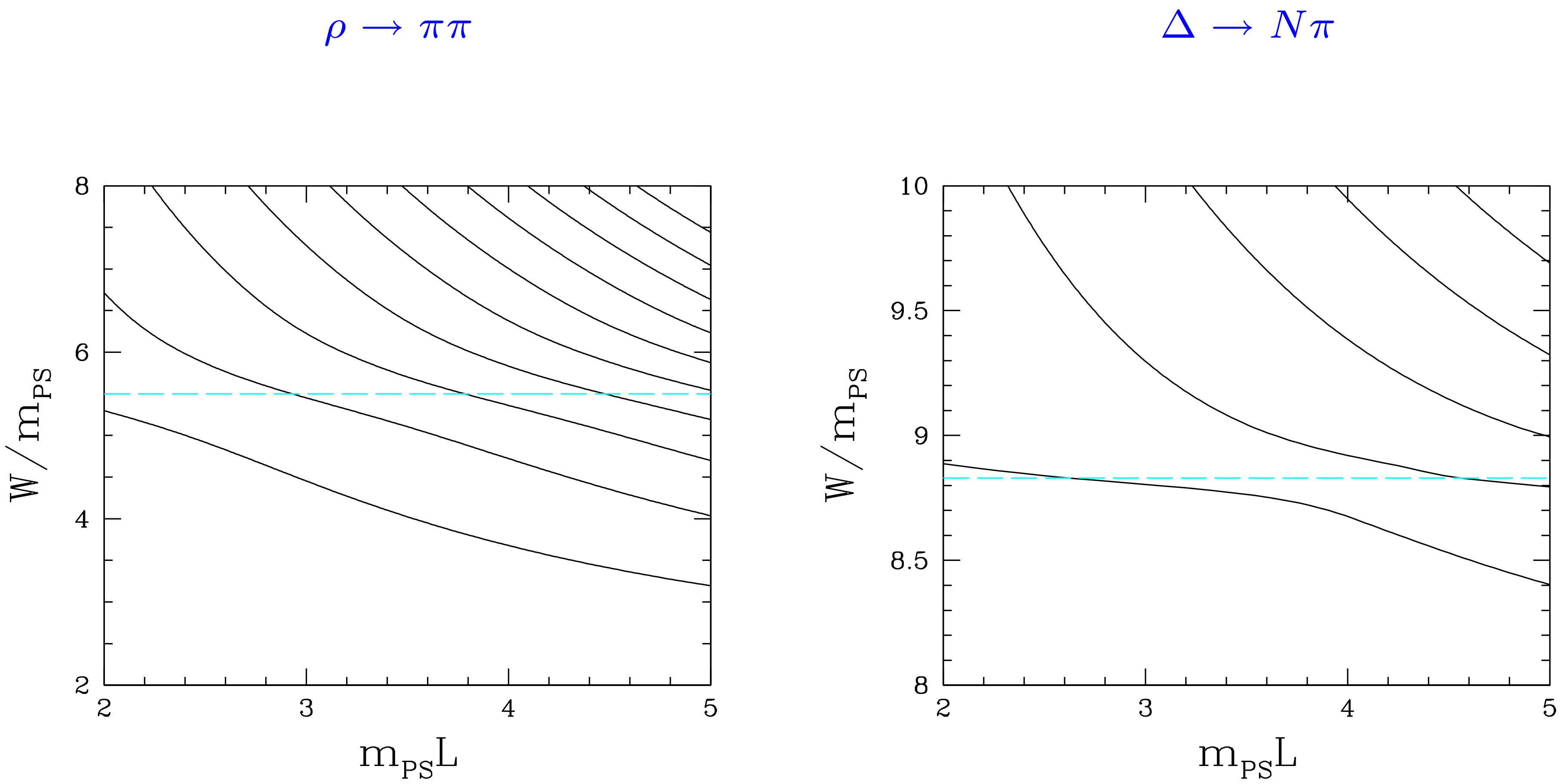
Clover Fermions

Advantages

- Local
- Transfer matrix
- $O(a)$ improved
- Flavor symmetry

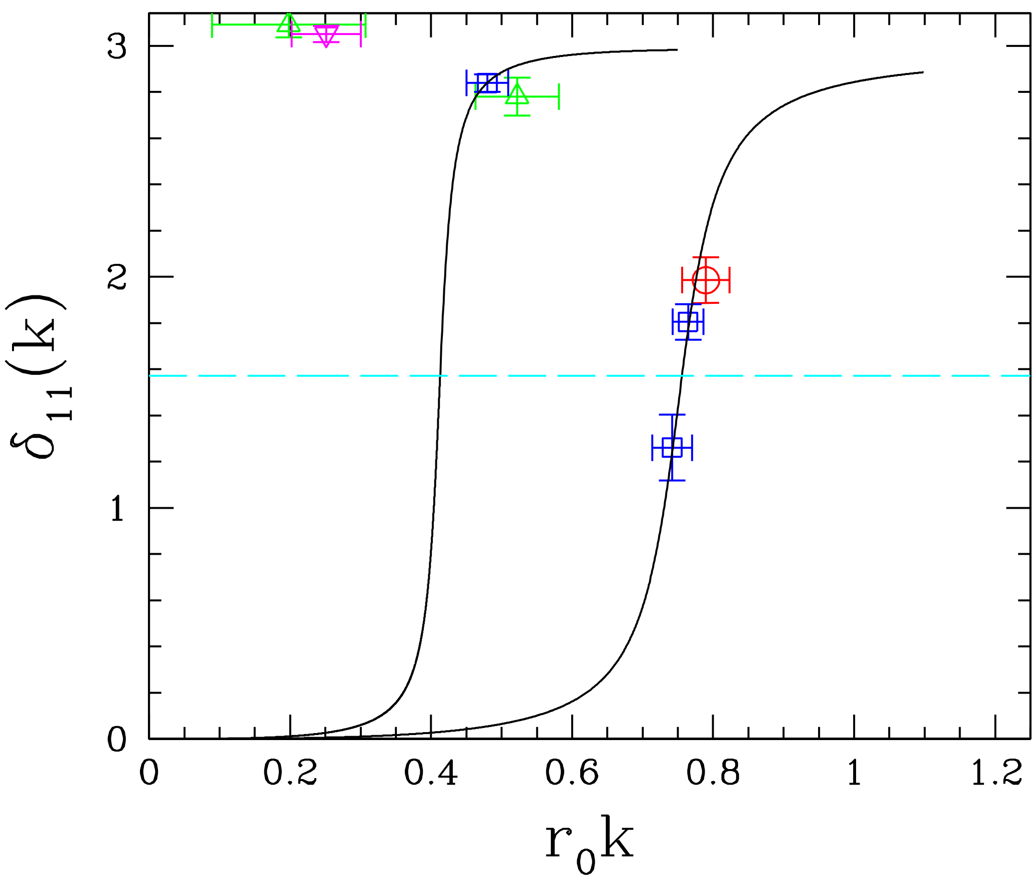
  Prerequisite to making contact with $SU(2)$ ChPT
  - Finite size corrections
  - Chiral extrapolation
  - Determination of low-energy constants
- Fast to simulate

β	$\kappa_{\rm sea}$	Volume	a [fm]	m_{PS} [MeV]
5.20	0.13420	$16^3 \times 32$	0.090	1010
5.20	0.13500	$16^3 \times 32$	0.090	830
5.20	0.13550	$16^3 \times 32$	0.090	620
5.25	0.13460	$16^3 \times 32$	0.084	990
5.25	0.13520	$16^3 \times 32$	0.084	830
5.25	0.13575	$24^3 \times 48$	0.084	600
5.25	0.13600	$24^3 \times 48$	0.084	430
5.26	0.13450	$16^3 \times 32$	0.083	1010
5.29	0.13400	$16^3 \times 32$	0.080	1170
5.29	0.13500	$16^3 \times 32$	0.080	930
5.29	0.13550	$24^3 \times 48$ $16^3 \times 32$ $12^3 \times 32$	0.080	810
5.29	0.13590	$24^3 \times 48$ $16^3 \times 32$ $12^3 \times 32$	0.080	590
5.29	0.13605	$24^3 \times 48$	0.080	480
5.29	0.13620	$24^3 \times 48$	0.080	390
5.29	0.13632	$40^3 \times 64$ $32^3 \times 64$ $24^3 \times 48$	0.080	250
In progress $\longrightarrow$ 5.29	0.13640	$64^3 \times 96$ $40^3 \times 64$	0.080	140
5.40	0.13500	$24^3 \times 48$	0.072	810
5.40	0.13560	$24^3 \times 48$	0.072	770
5.40	0.13610	$24^3 \times 48$	0.072	610
5.40	0.13625	$24^3 \times 48$	0.072	530
5.40	0.13640	$32^3 \times 64$	0.072	420
5.40	0.13660	$32^3 \times 64$	0.072	240



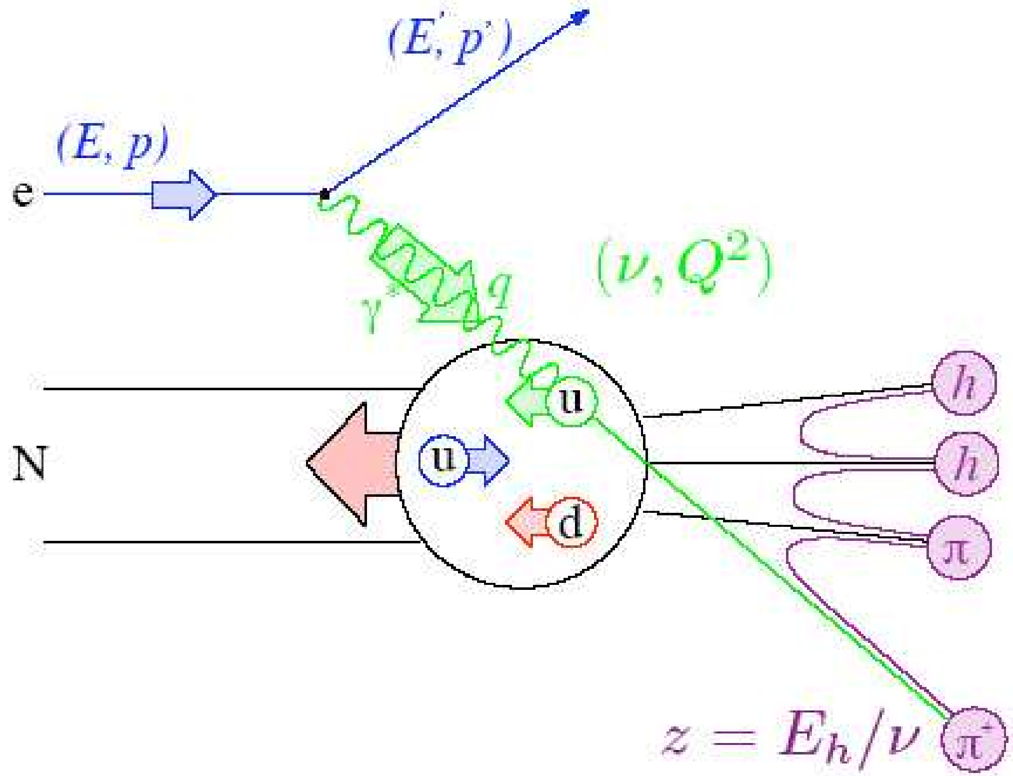
ρ → ππ
Δ → Nπ
W/m_PS
m_PS L
2
3
4
5
6
8
8.5
9
9.5
10

$\rho \longrightarrow \pi\pi$

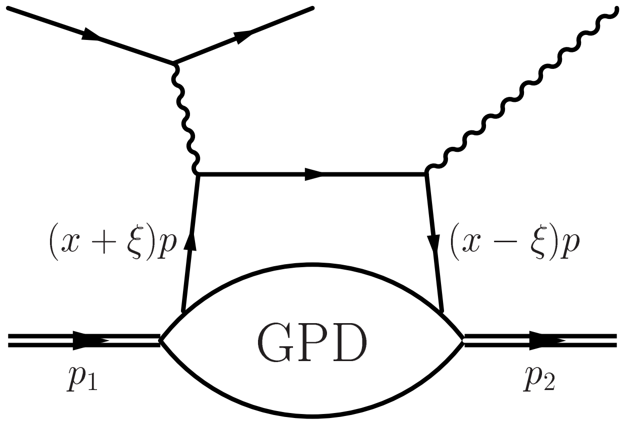


Basics

Inclusive



OPE



$p = \frac{1}{2}(p_1 + p_2)$, $\Delta = p_2 - p_1$, $q = \frac{1}{2}(q_1 + q_2)$

$\xi = 0$: Momentum transfer of the struck parton purely transverse, i.e. $\Delta = \mathbf{\Delta}_\perp$

$$J(q)\, J(-q) \;=\; \sum_n c_n \;\times\; \left\{ \begin{array}{l} \mathcal{O}^q_{\mu_1 \cdots \mu_n} = \left(\frac{i}{2}\right)^{n-1} \bar{q}\gamma_{\mu_1} \overleftrightarrow{D}_{\mu_2} \cdots \overleftrightarrow{D}_{\mu_n} q \\ \mathcal{O}^{5q}_{\sigma\mu_1 \cdots \mu_n} = \left(\frac{i}{2}\right)^{n} \bar{q}\gamma_{\sigma}\gamma_5 \overleftrightarrow{D}_{\mu_1} \cdots \overleftrightarrow{D}_{\mu_n} q \\ \mathcal{O}^{Tq}_{\mu\nu\mu_1 \cdots \mu_n} = \left(\frac{i}{2}\right)^{n} \bar{q}\sigma_{\mu\nu}\gamma_5 \overleftrightarrow{D}_{\mu_1} \cdots \overleftrightarrow{D}_{\mu_n} q \end{array} \right.$$

Nucleon

$$\langle p_1, s|\, \mathcal{O}^q_{\{\mu_1\cdots\mu_n\}}\, |p_2, s\rangle = \bar{u}(p_1, s)\Big[A^q_n(\Delta^2)\ \gamma_{\{\mu_1}$$

$$+ B^q_n(\Delta^2)\ \frac{\mathrm{i}\Delta^\alpha}{2m_N}\sigma_{\alpha\{\mu_1}\Big] p_{\mu_2}\cdots p_{\mu_n\}}\, u(p_2, s)\ +\ \cdots$$

$$\langle p_1, s|\mathcal{O}^{5q}_{\{\mu\mu_1\cdots\mu_n\}}|p_2, s\rangle = \bar{u}(p_1, s)\Big[\tilde{A}^q_{n+1}(\Delta^2)\ \gamma_{\{\mu}\gamma_5 p_{\mu_1}\cdots p_{\mu_n\}}\Big]u(p_2, s)\ +\ \cdots$$

$$\langle p_1, s|\mathcal{O}^{Tq}_{\mu\{\nu\mu_1\cdots\mu_n\}}|p_2, s\rangle = \bar{u}(p_1, s)\Big[A^{Tq}_{n+1}(\Delta^2)\ \sigma_{\mu\{\nu}\gamma_5 - \tilde{A}^{Tq}_{n+1}(\Delta^2)\big(\frac{\Delta^2}{2m_N^2}\sigma_{\mu\{\nu} - \frac{\Delta_\mu\Delta_\alpha}{2m_N^2}\sigma_{\alpha\{\nu}\big)\gamma_5$$

$$+ \bar{B}^{Tq}_{n+1}(\Delta^2)\ \epsilon_{\alpha\beta\mu\{\nu}\frac{\Delta_\alpha\gamma_\beta}{2m_N}\Big]\ p_{\mu_1}\cdot\cdot p_{\mu_n\}}u(p_2, s)\ +\ \cdots$$

$$A_n^q(\Delta^2) = \int_0^1 dx\, x^{n-1} H^q(x, \Delta^2)$$

$$H^q(x,0) = q(x)$$

$$B_n^q(\Delta^2) = \int_0^1 dx\, x^{n-1} E^q(x, \Delta^2)$$

$$\tilde{A}_n^q(\Delta^2) = \int_0^1 dx\, x^{n-1} \tilde{H}^q(x, \Delta^2)$$

$$\tilde{H}^q(x,0) = \Delta q(x)$$

$$A_n^{Tq}(\Delta^2) = \int_0^1 dx\, x^{n-1} H^{Tq}(x, \Delta^2)$$

$$H^{Tq}(x,0) = \delta q(x)$$

↑ GFFs

↑ GPDs

$$\boxed{\frac{1}{2}\Big(A_2^q(0) + B_2^q(0)\Big) = J^q}$$

Ji

$$A_1^q\ (\Delta^2) = F_1^q(\Delta^2)$$

$$B_1^q\ (\Delta^2) = F_2^q(\Delta^2)$$

$$\tilde{A}_1^q\ (\Delta^2) = g_A^q\ (\Delta^2)$$

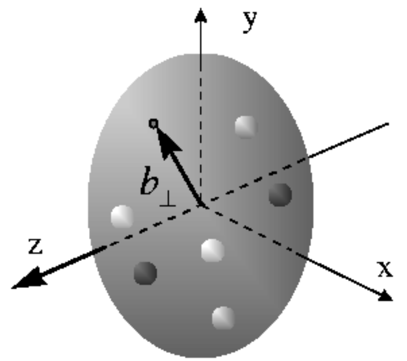
$$A_1^{Tq}(\Delta^2) = g_T^q\ (\Delta^2)$$

$$\Delta^2 = t = -Q^2$$

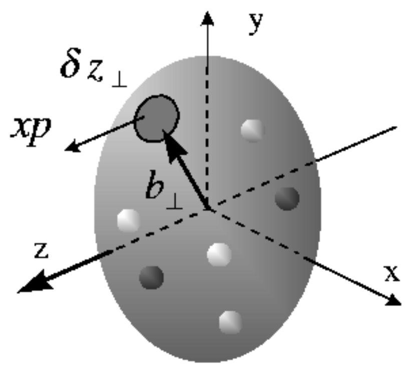
Impact Parameter Space

Generically

$$A_n^q(\mathbf{b}_\perp^2) = \int \frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2} \mathrm{e}^{\,\mathrm{i}\,\mathbf{b}_\perp \boldsymbol{\Delta}_\perp} A_n^q(\boldsymbol{\Delta}_\perp^2)$$



$$H^q(x, \mathbf{b}_\perp^2) = \int \frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2} \mathrm{e}^{\,\mathrm{i}\,\mathbf{b}_\perp \boldsymbol{\Delta}_\perp} H^q(x, \boldsymbol{\Delta}_\perp^2)$$



Probability interpretation

Burkardt

$$H^q(x, \Delta^2) = \int_x^1 \frac{dy}{y} C\left(\frac{x}{y}, \Delta^2\right) q(y)$$

Similarly for $\tilde{H}^q$ and $H^{T\,q}$

$$\int_0^1 dx\, x^n C(x, \Delta^2) = \frac{A_{n+1}(\Delta^2)}{A_{n+1}(0)}$$

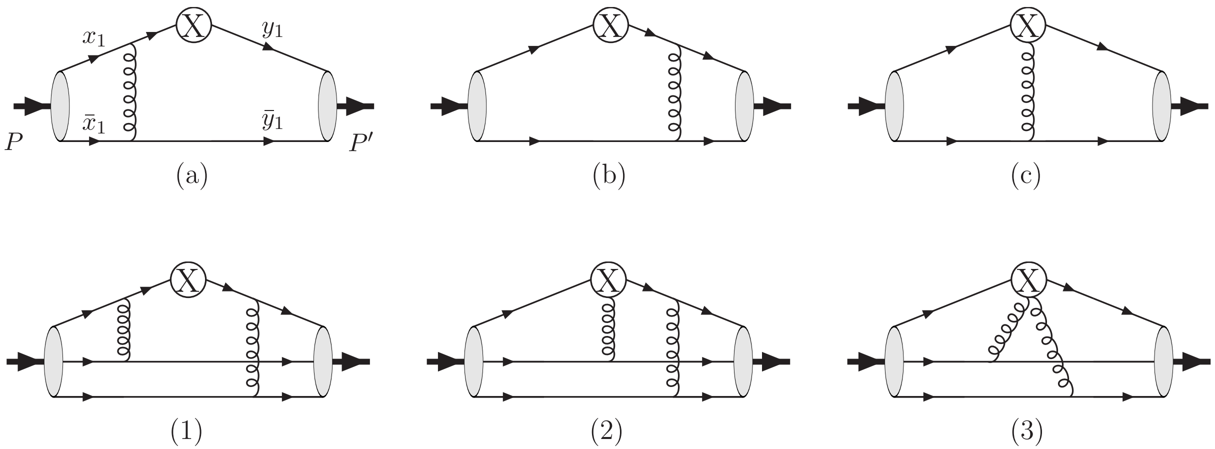
$E.g.\quad \dfrac{1}{(1 - \Delta^2/M_n^2)^p}$

By inverse Mellin transform

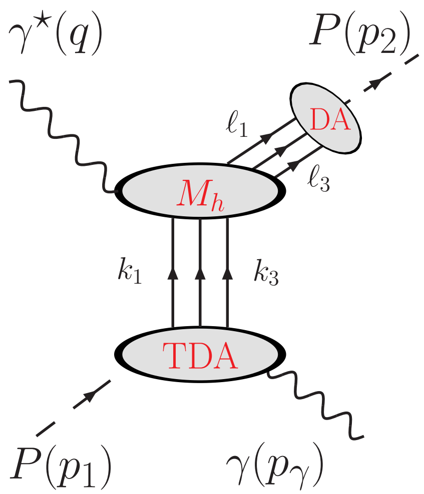
$$H^q(x, \mathbf{b}_\perp^2) = \int_x^1 \frac{dy}{y} C\left(\frac{x}{y}, \mathbf{b}_\perp^2\right) q(y)$$

$$C(x, \mathbf{b}_\perp^2) = \int \frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2} e^{i\,\mathbf{b}_\perp \boldsymbol{\Delta}_\perp} C(x, \boldsymbol{\Delta}_\perp^2)$$

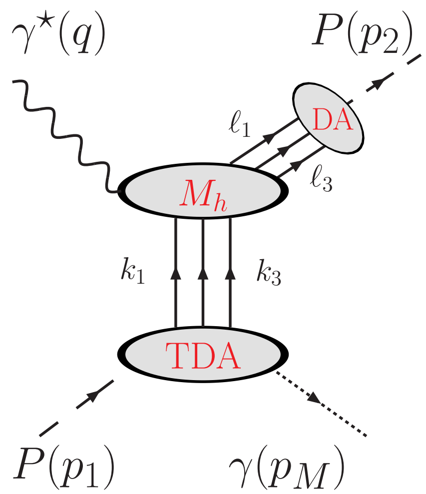
Exclusive



x_1
y_1
$\bar{x}_1$
$\bar{y}_1$
P
P'
X
(a)
(b)
(c)
(1)
(2)
(3)



$\gamma^\star(q)$
$P(p_2)$
ℓ_1
DA
ℓ_3
M_h
k_1
k_3
TDA
$P(p_1)$
$\gamma(p_\gamma)$



$\gamma^\star(q)$
$P(p_2)$
ℓ_1
DA
ℓ_3
M_h
k_1
k_3
TDA
$P(p_1)$
$\gamma(p_M)$

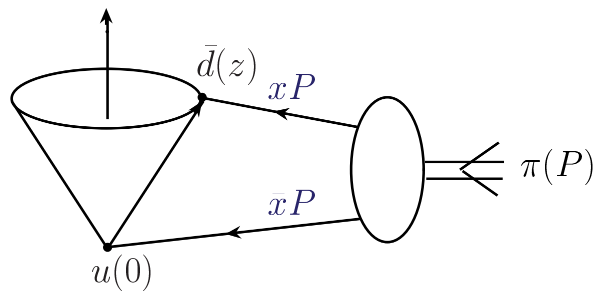
Distribution Amplitudes Leading Twist

Pion

$$\langle 0|\bar{d}(z)\gamma_\mu\gamma_5 U(z,0)u(0)|\pi(p)\rangle$$

$$= i f_\pi p_\mu \int_0^1 dx\, e^{ix(zp)}\phi_\pi(x,\mu^2)$$

Projection onto LC coordinates



$$\phi_\pi(x,\mu^2) = \int_{|k_\perp|<\mu} d^2k_\perp\, \phi_{BS}(x,k_\perp,\mu^2)$$

Nucleon

$$\langle 0|\left(u_a^\uparrow(z_1)C\not{p}\,u_b^\downarrow(z_2)\right)\not{p}\,d_c^\uparrow(z_3)U_{aa'}(z_1,z_0)U_{bb'}(z_2,z_0)U_{cc'}(z_3,z_0)\epsilon_{a'b'c'}\,|p\rangle$$

$$= \frac{f_N}{4}\not{p}N^\uparrow \int [dx]\, e^{i\sum x_i(z_i p)}\phi_N(x_1,x_2,x_3,\mu^2)$$

Braun et al.

Generic moment

$$\langle 0|\mathcal{O}_{\rho\,\lambda_1\cdots\lambda_l\,\mu_1\cdots\mu_m\,\nu_1\cdots\nu_n\;\alpha}(0)|p\rangle \;=\; \phi_N^{lmn}(\mu^2)\,p_\rho\,p_{\lambda_1}\cdots p_{\lambda_l}\,p_{\mu_1}\cdots p_{\mu_m}\,p_{\nu_1}\cdots p_{\nu_n}\,N_\alpha^{\uparrow}(p)$$

$$\phi_N^{lmn}(\mu^2) = \int [dx]\; x_1^l\, x_2^m\, x_3^n\; \phi_N(x_1, x_2, x_3, \mu^2)$$

$$\mathcal{O}_{\rho\,\lambda_1\cdots\lambda_l\,\mu_1\cdots\mu_m\,\nu_1\cdots\nu_n\;\alpha}(0) \;=\; \Big([i^l D_{\lambda_1}\cdots D_{\lambda_l}\,u^{\uparrow}(0)]_a\; C\gamma_\rho\;[i^m D_{\mu_1}\cdots D_{\mu_m}\,u^{\downarrow}(0)]_b\Big)$$

$$\times\;\; [i^n D_{\nu_1}\cdots D_{\nu_n}\,d^{\uparrow}(0)]_{c\;\alpha}\;\epsilon_{abc}$$

Applications Two Examples

Form factor

$$G_M(\Delta^2) = \int [dy][dz]\, \phi_N^*(y_1, y_2, y_3, \mu^2)\, T_M([y], [z], \Delta^2)\, \phi_N(z_1, z_2, z_3, \mu^2) + \text{HT}$$

Δ^2 large

$\uparrow$

Perturbative above scale Δ^2

DVCS

$$H^q(x, \Delta^2, \xi) = \int [dy][dz]\, \phi_N^*(y_1, y_2, y_3, \mu^2)\, T_H^q([y], [z], x, \Delta^2, \xi)\, \phi_N(z_1, z_2, z_3, \mu^2) + \text{HT}$$

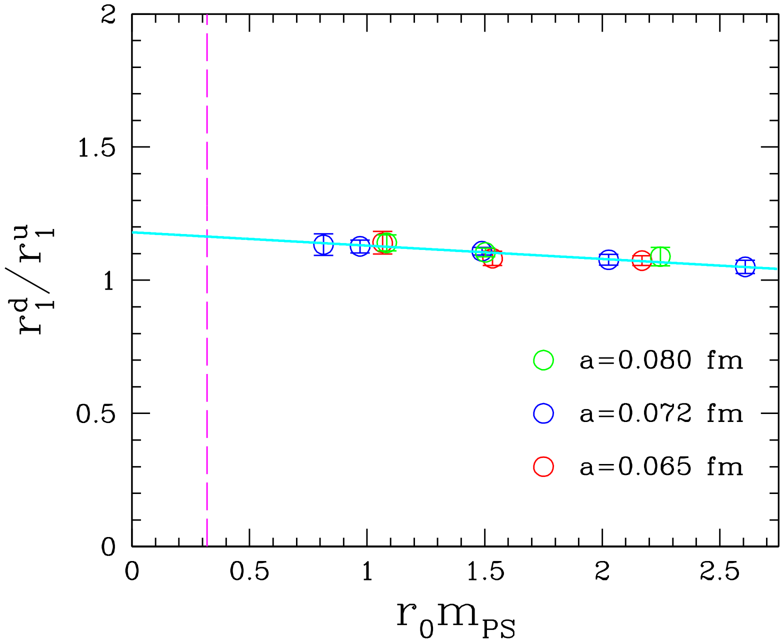
Δ^2 large

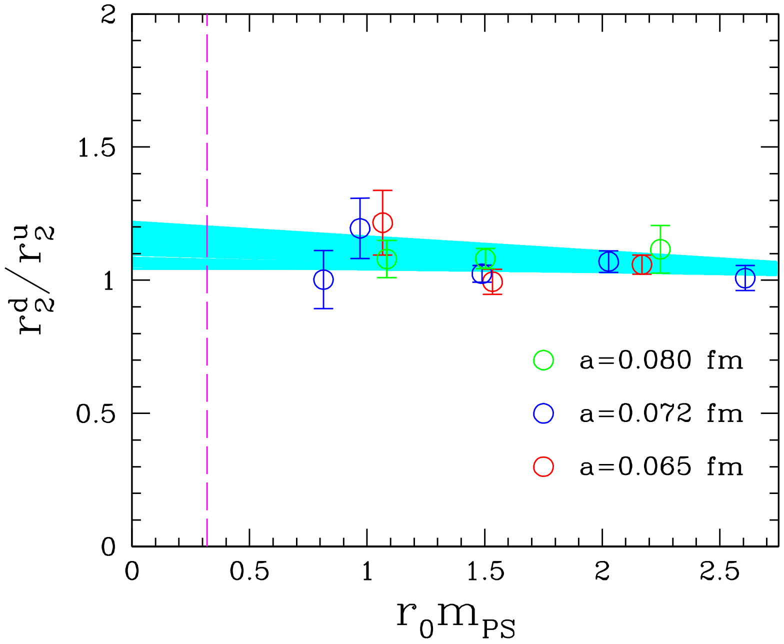
Hoodbhoy, Ji & Yuan

Nucleon

Form Factors

$$F_i(Q^2) = F_i(0)\big(1 - \frac{1}{6} r_i^2 Q^2 + O(Q^4)\big) \qquad i = 1, 2$$

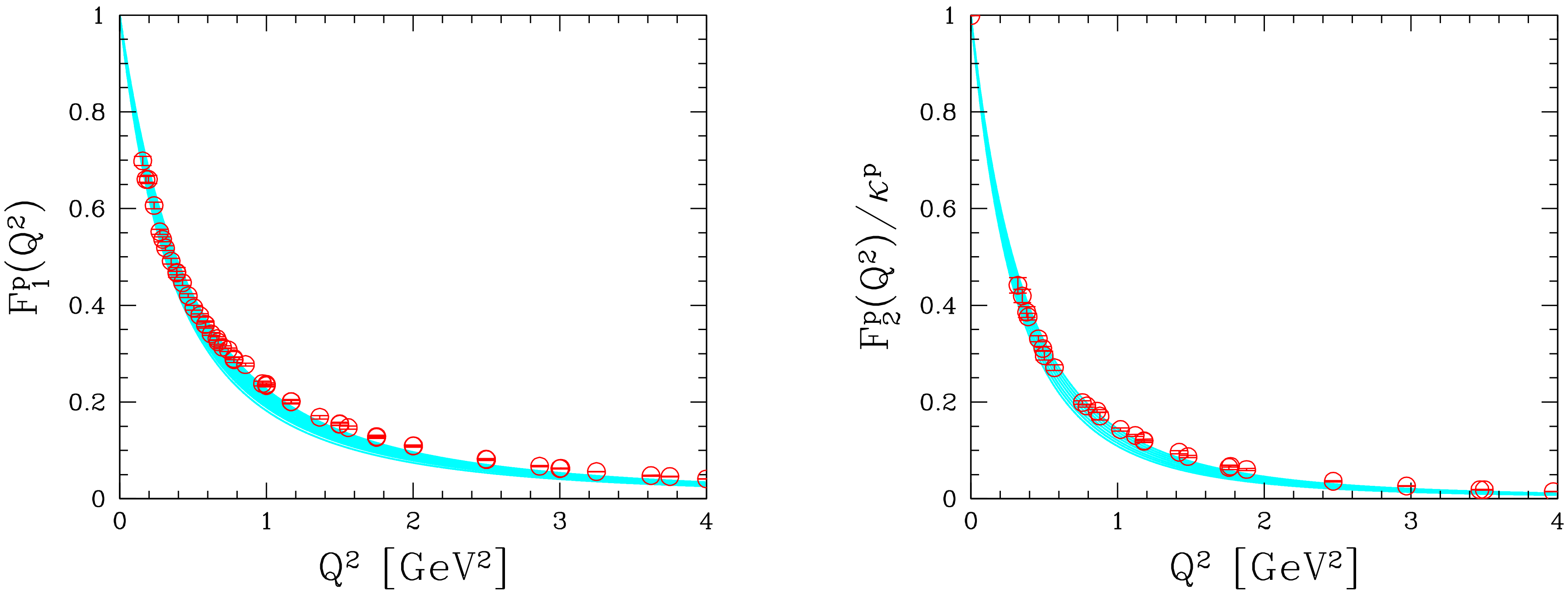




Proton

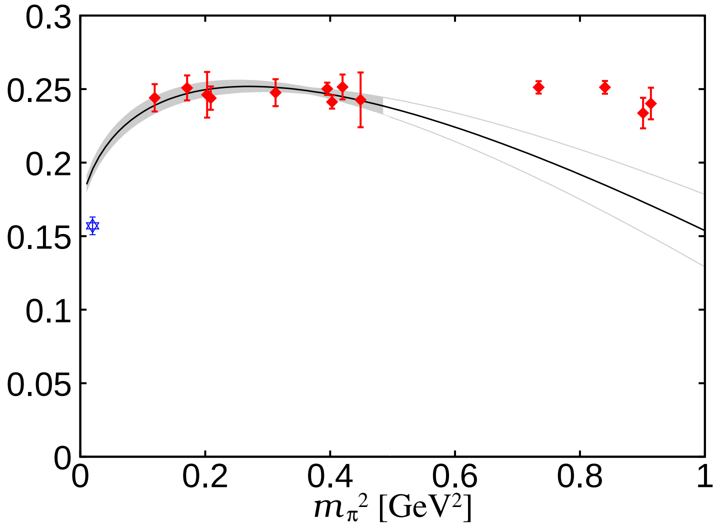
$r^d_{1,2} > r^u_{1,2}$

Comparison with experiment



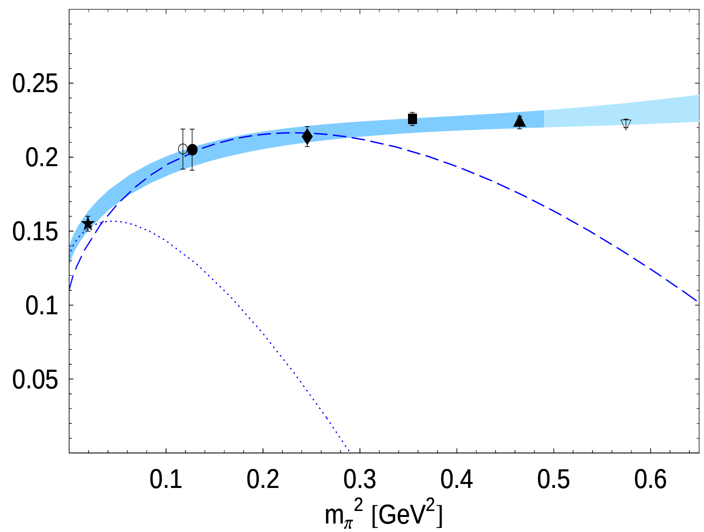
Moments of Structure Functions

$$\langle x \rangle = \int dx\, x\left[u(x,Q^2) - d(x,Q^2)\right]$$



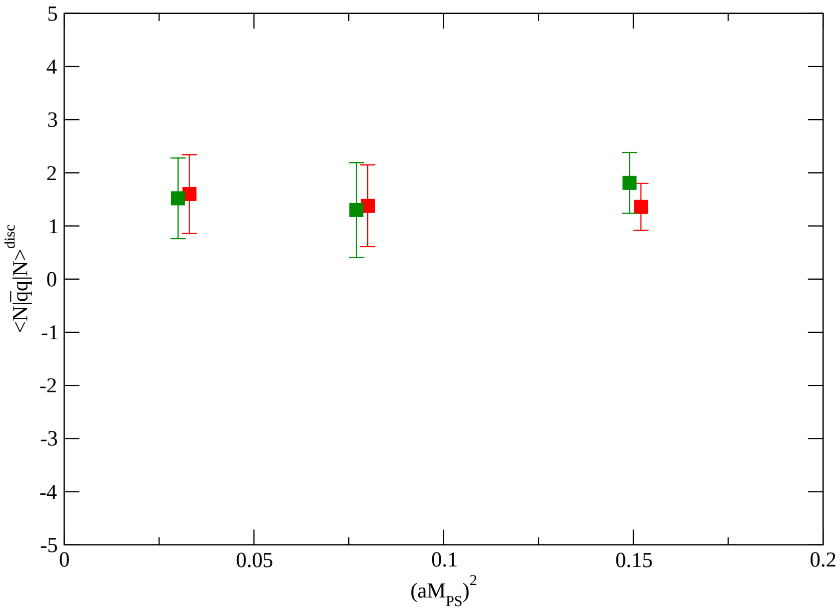
$Q^2 = 4$ GeV^2

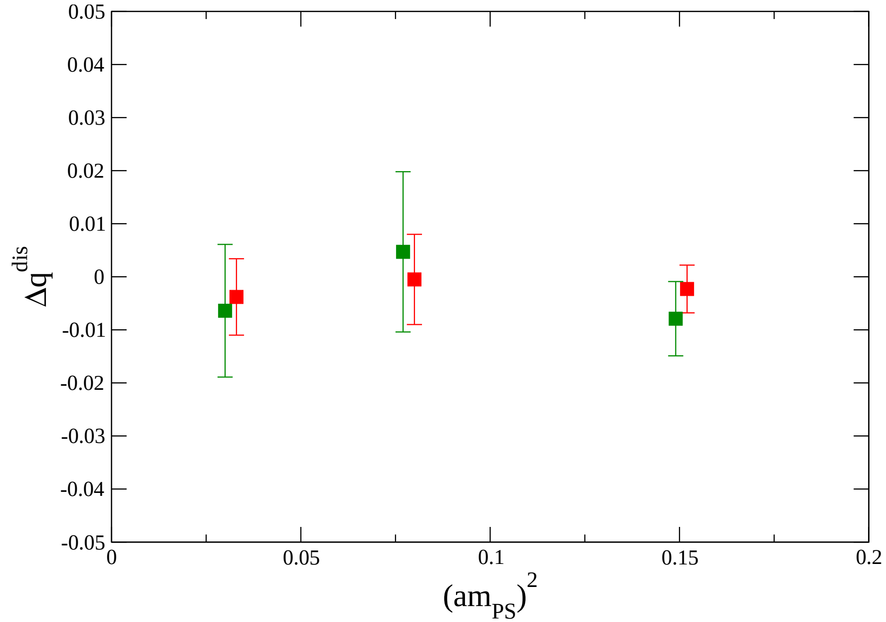
QCDSF



LHP

Disconnected





Bali, Collins & Schäfer

Spin Asymmetries

Transverse spin density

$\Downarrow$

$\lambda_\perp$ quark spin

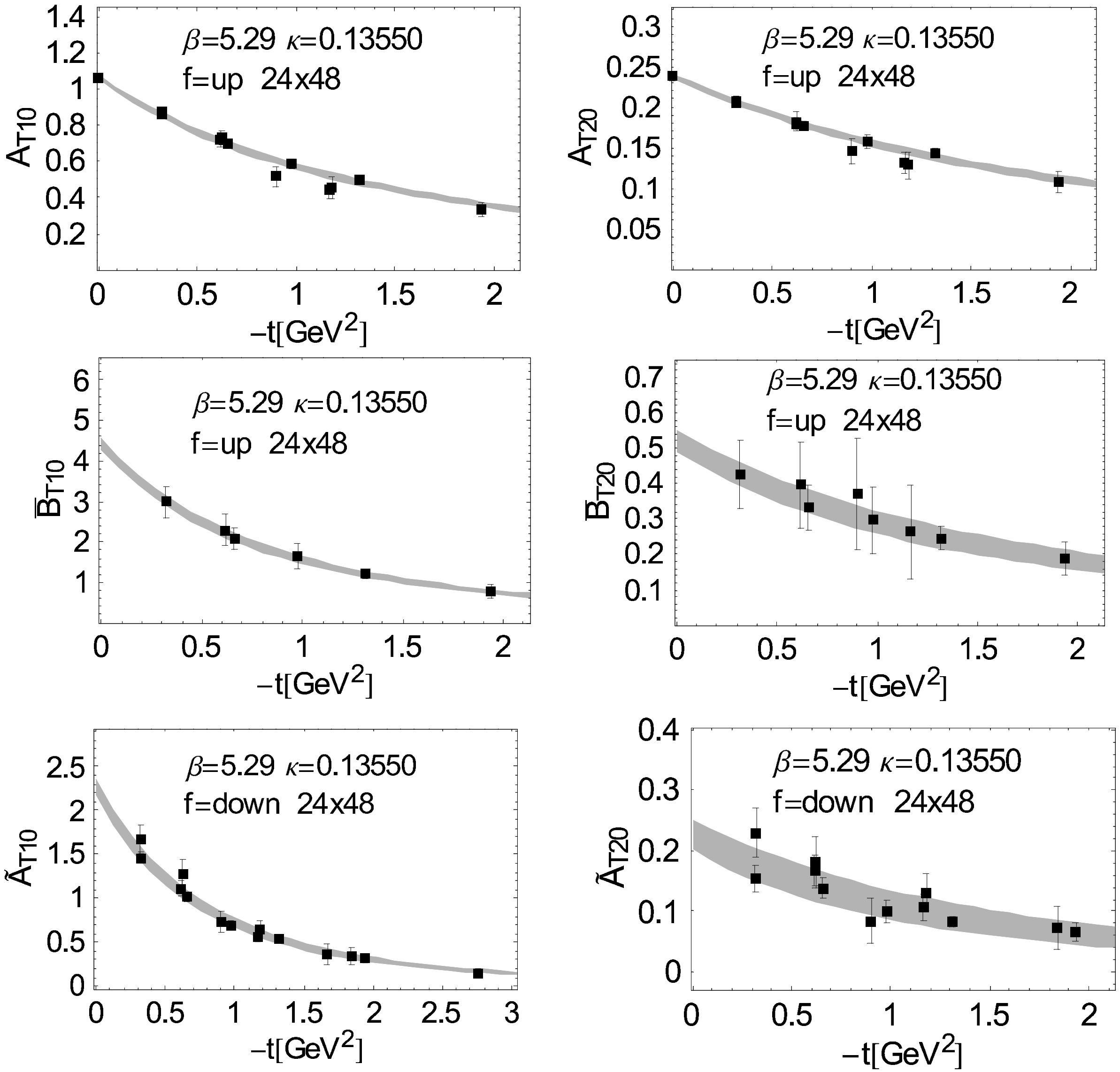
$s_\perp$ nucleon spin

$$\langle p_+, s_\perp | \bar{q}(\mathbf{b}_\perp) \left[\gamma_+ - \lambda_{\perp i}\, \sigma_{+j}\, \gamma_5\right] q(\mathbf{b}_\perp) | p_+, s_\perp \rangle = \Big\{ A_1^q(\mathbf{b}_\perp^2) + \lambda_{\perp i}\, s_{\perp i} \Big[A_1^{Tq}(\mathbf{b}_\perp^2) - \frac{1}{4m_N^2} \Delta_{b_\perp} \tilde{A}_1^{Tq}(\mathbf{b}_\perp^2) \Big] - \frac{1}{m_N} \epsilon_{ij} b_{\perp j} \Big[s_{\perp i} B_1^q(\mathbf{b}_\perp^2)' + \lambda_{\perp i}\, \bar{B}_1^{Tq}(\mathbf{b}_\perp^2)' \Big] + \frac{1}{m_N^2} \lambda_{\perp i} \left(2 b_{\perp i}\, b_{\perp j} - \mathbf{b}_\perp^2 \delta_{ij}\right) s_{\perp j}\, \tilde{A}_1^{Tq}(\mathbf{b}_\perp^2)'' \Big\}$$

$\Uparrow$

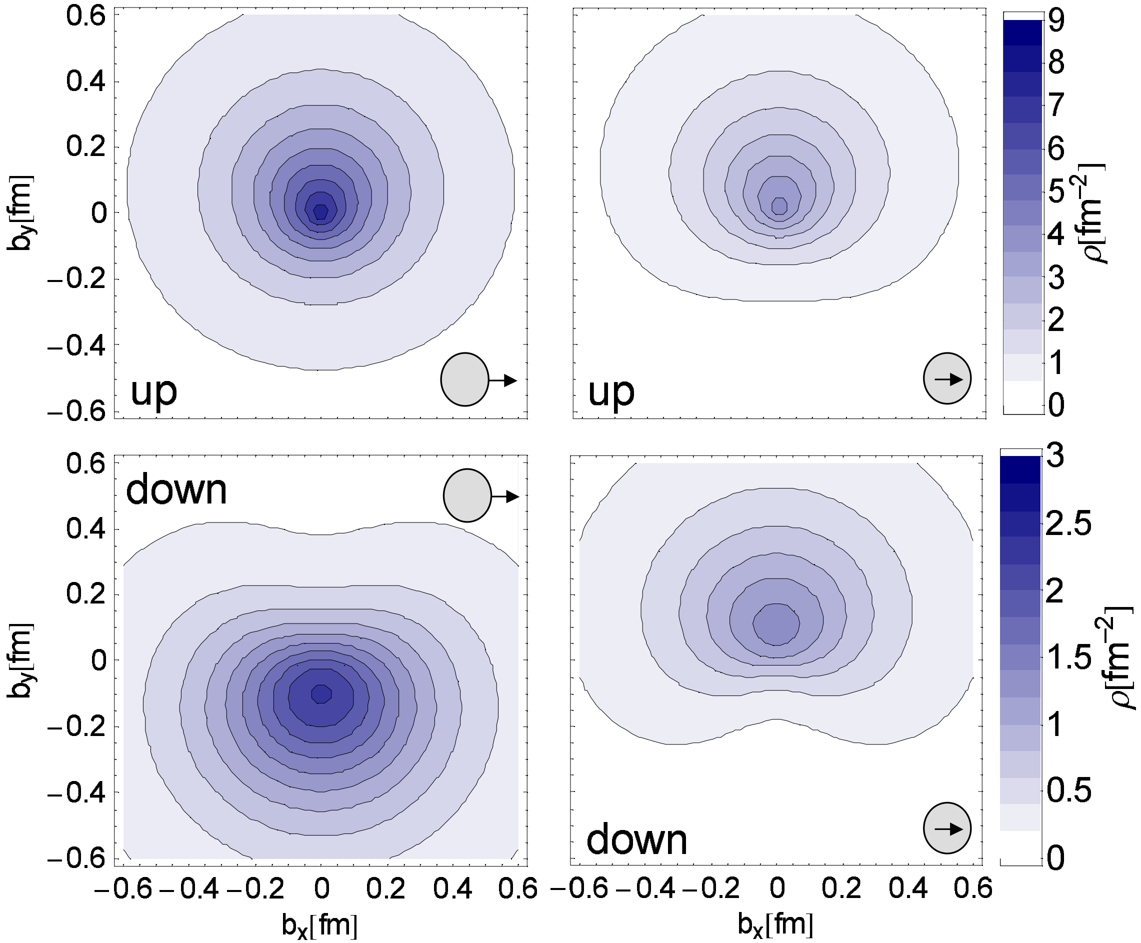
Quadrupole

Diehl & Hägler



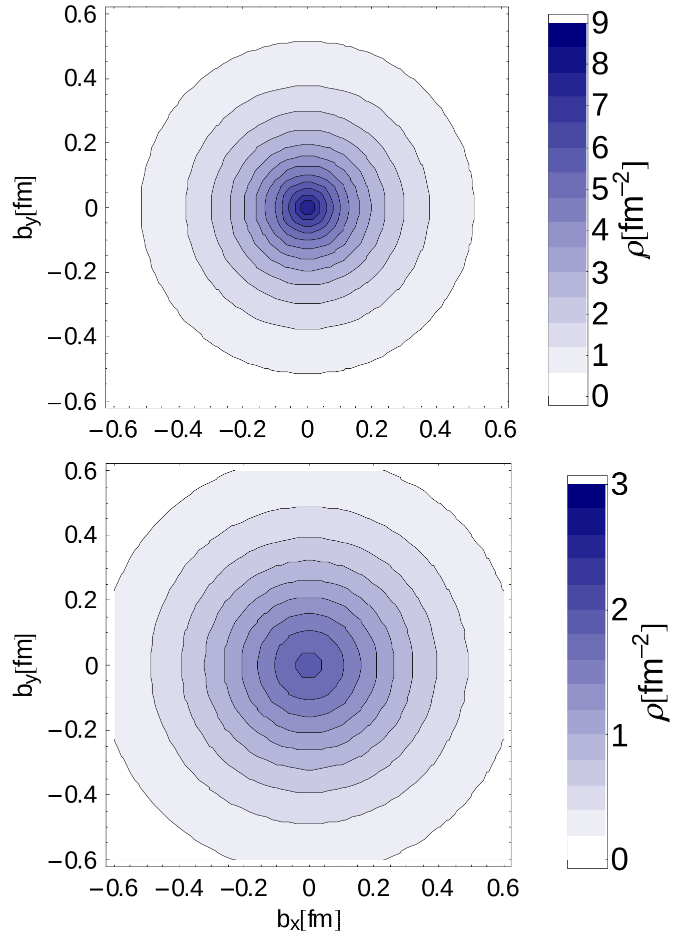
Dipole fit $\quad$ 1^{st} moment

To be extrapolated to chiral limit



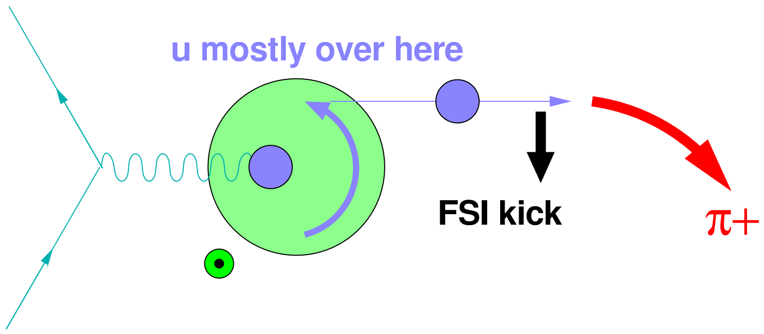
Sivers effect

Boer-Mulders effect



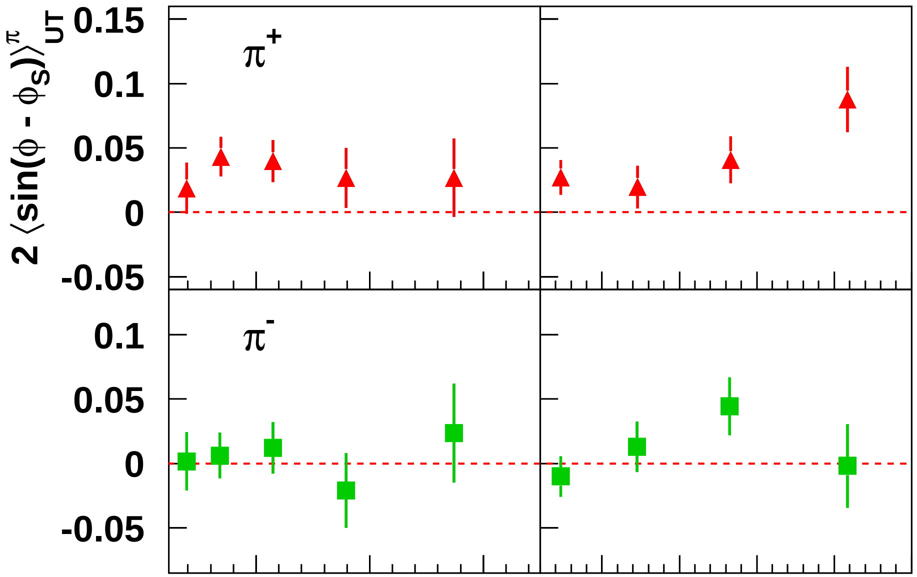
Unpolarized

$$r_T^d > r_T^u$$



Courtesy of G. Schnell

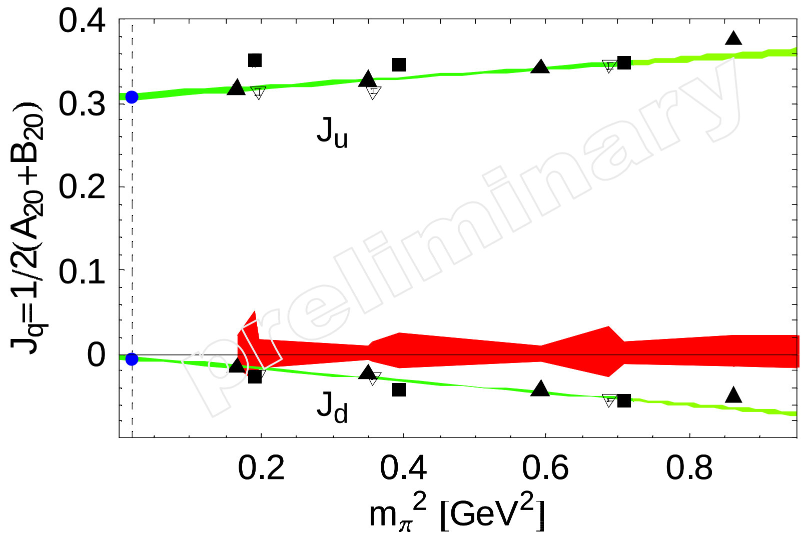
Sivers effect

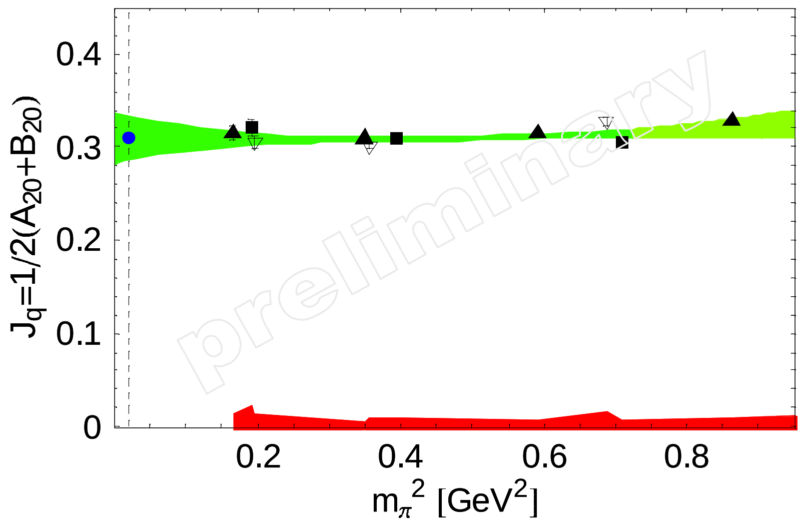


Hermes

Sivers effect $\Leftarrow$ Orbital angular momentum

$$J^q = \frac{1}{2}\left(A_2^q(0) + B_2^q(0)\right) \equiv \frac{1}{2}\Delta\Sigma^q + L^q$$

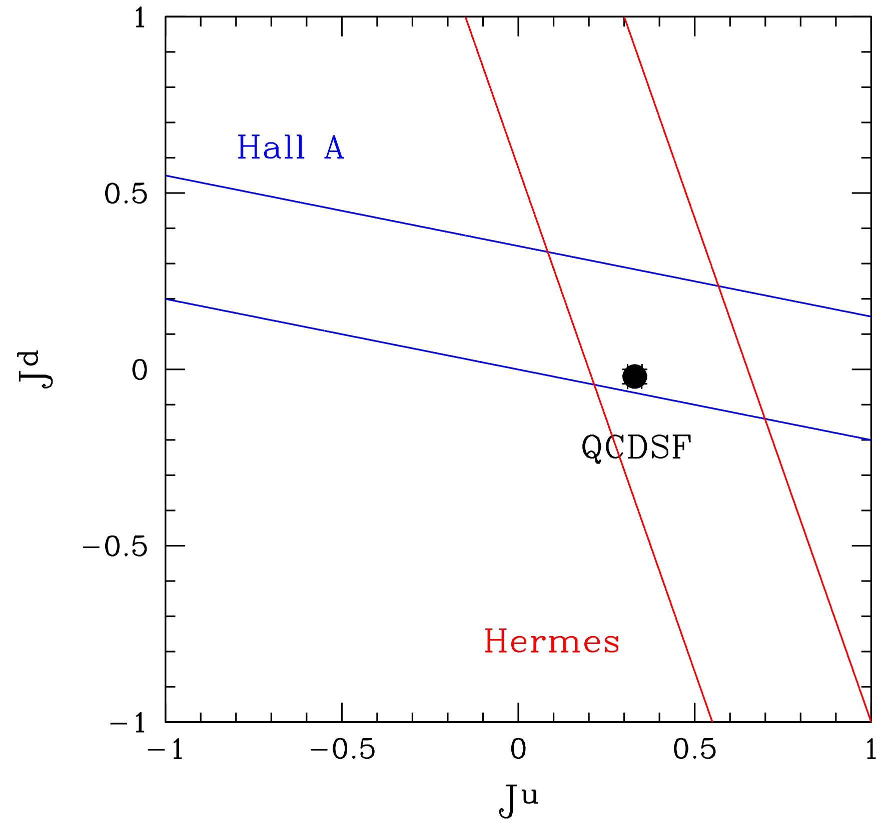




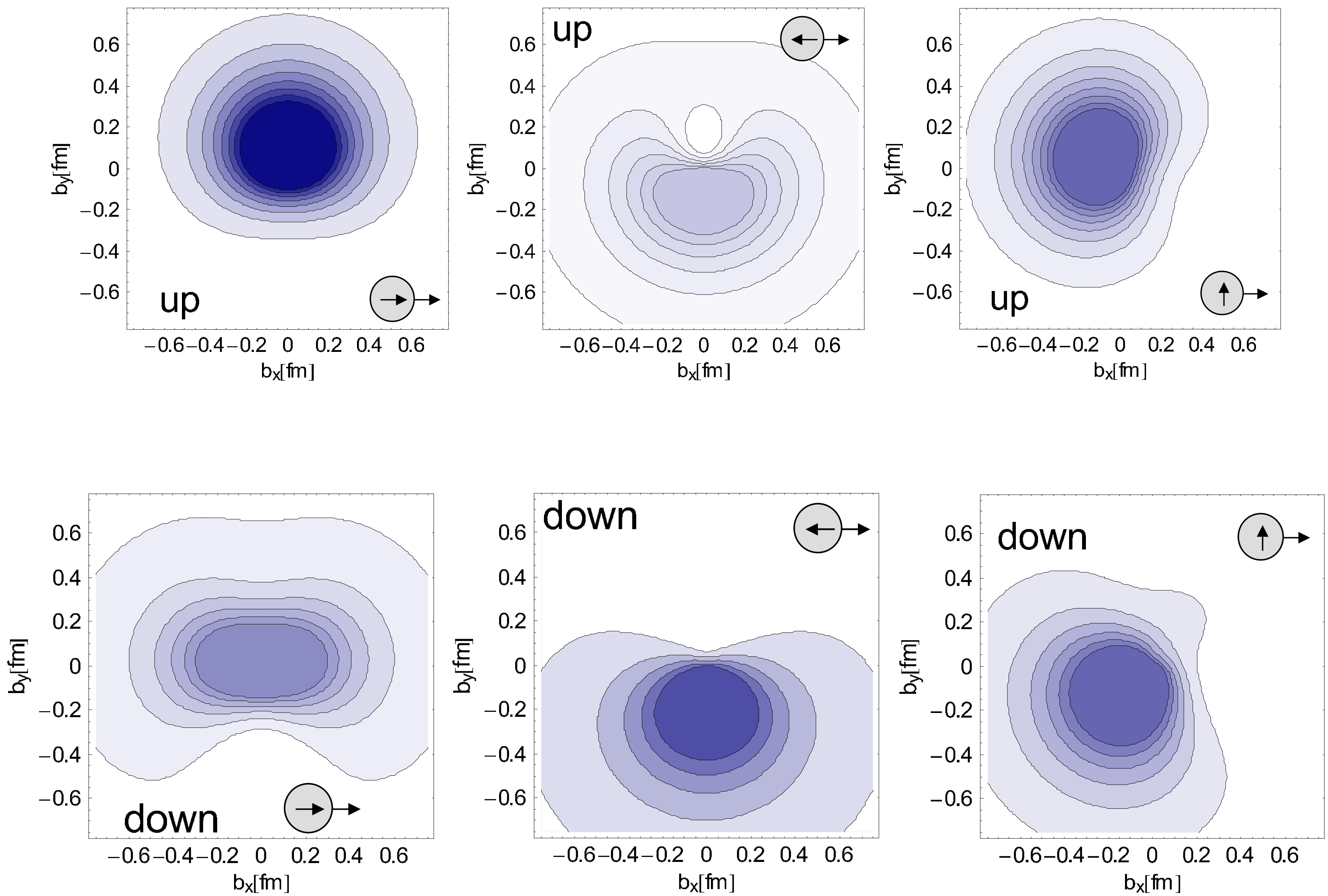
ChPT fit Chen & Ji

$J^u = 0.33(2)$ $J^d = -0.02(2)$ $L^u = -0.13(4)$ $L^d = 0.15(4)$

Experiment



Nucleon and quarks both polarized Spin-orbit coupling



Distribution Amplitudes

Expansion

$$\phi_N(x_1, x_2, x_3, \mu^2) = 120\, x_1 x_2 x_3 \sum_{n=0}^{\infty} \sum_{l=0}^{n} c_{nl}(\mu_0)\, P_{nl}(x_1, x_2, x_3)\, L^{\gamma_{nl}/\beta_0}$$

LLog

$$L = \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}$$

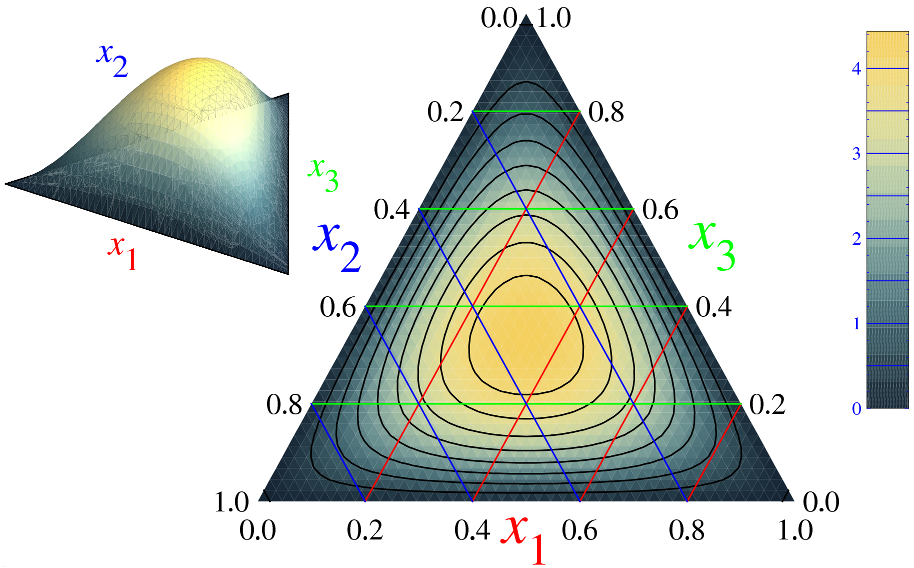
$$c_{10} = \frac{7}{2}\left[3\left(\phi_N^{100}(\mu_0^2) + \phi_N^{001}(\mu_0^2)\right) - 2\right]$$

$$c_{11} = \frac{63}{2}\left(\phi_N^{100}(\mu_0^2) + \phi_N^{001}(\mu_0^2)\right)$$

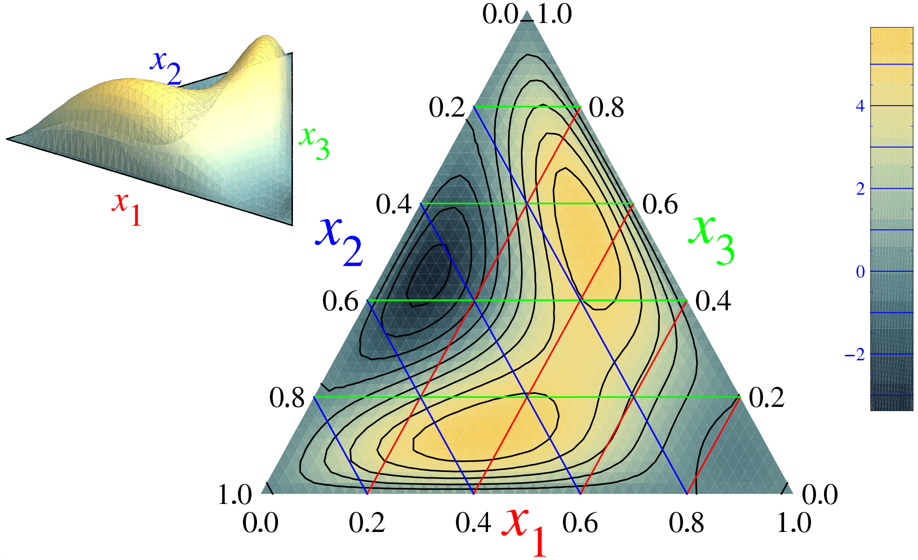
$$c_{21} = -\frac{126}{5}\left(\phi_N^{200}(\mu_0^2) + \phi_N^{002}(\mu_0^2) + 3\phi_N^{101}(\mu_0^2)\right) + \frac{18}{5}(4 + c_{10})$$

$$\vdots$$

$\phi_N(x_1, x_2, x_3, \mu^2)$ Barycentric contour plot



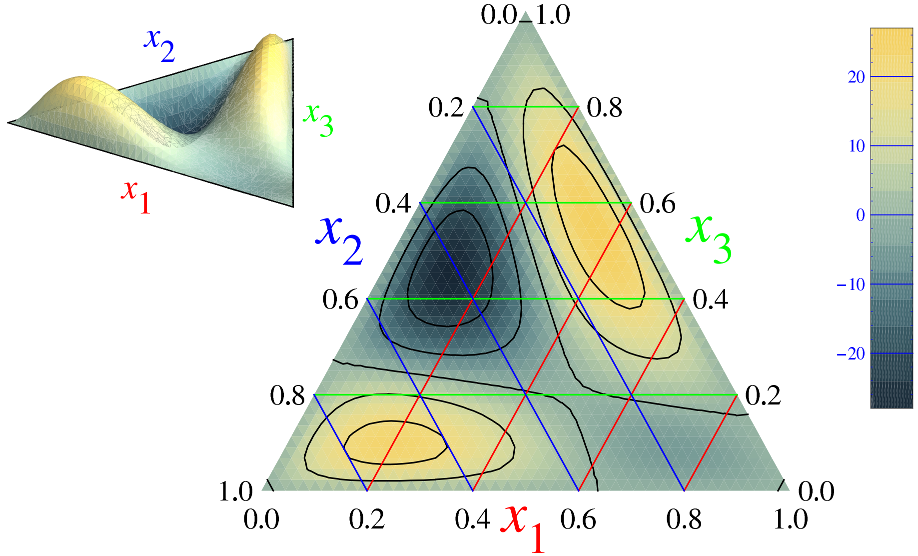
$\mu^2 \to \infty$

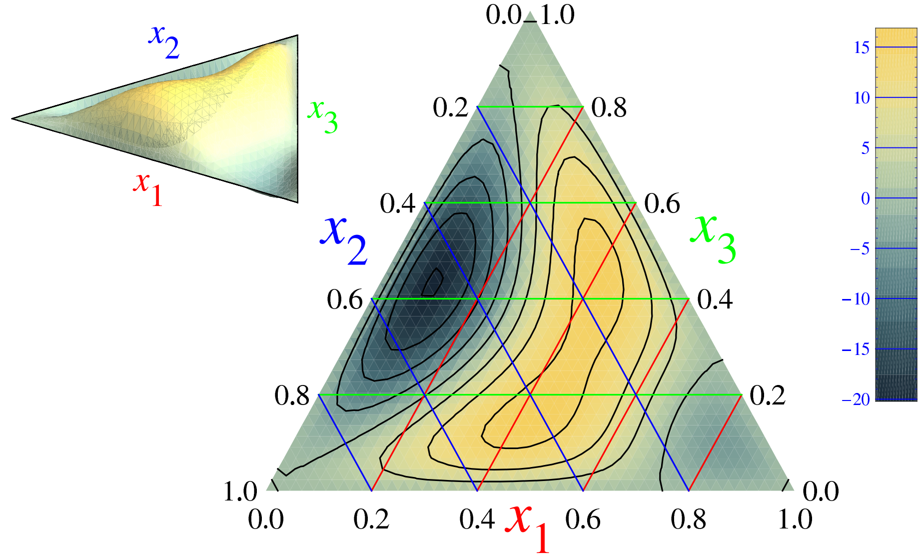


$\mu^2 = 4\ \text{GeV}^2$

Nucleon

$N^*(1535)$

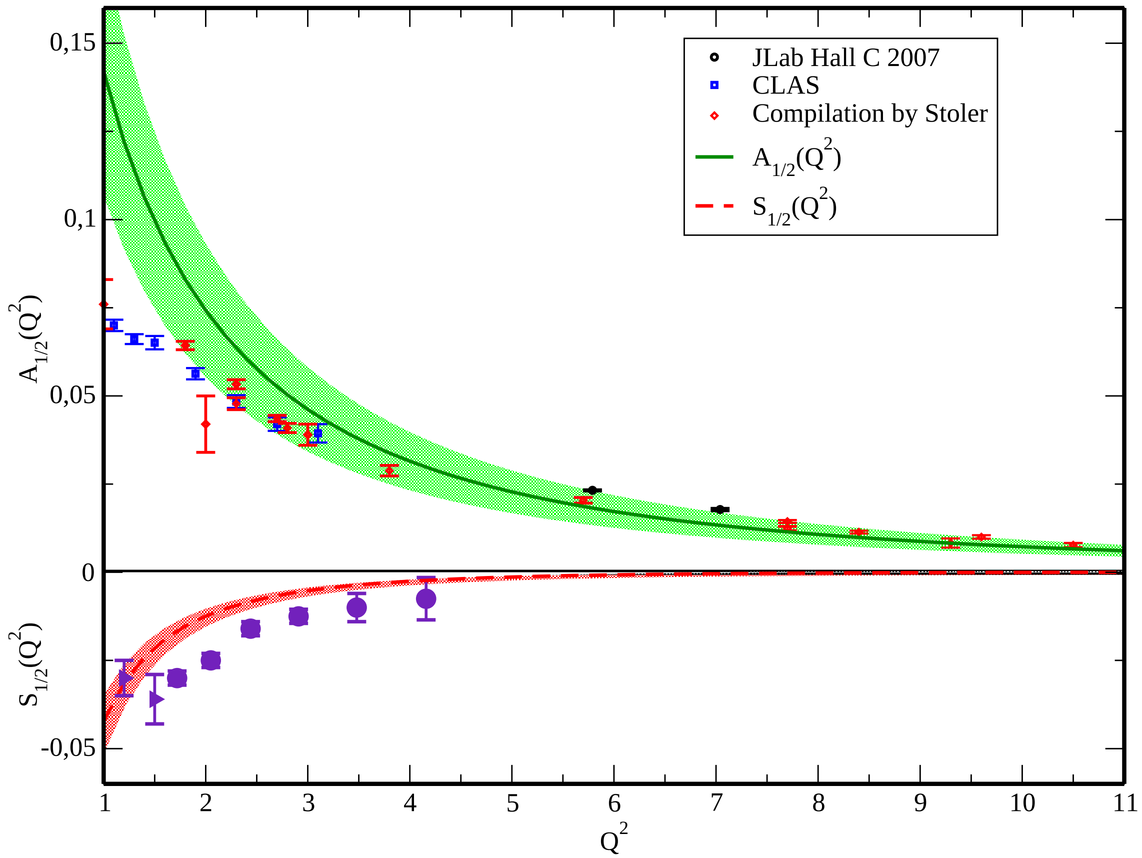




Strong correlation of $(u^{\downarrow}d^{\uparrow})$ diquark

$\mu^2 = 1\ \mathrm{GeV}^2$

$\gamma^* N \to N^*(1535)$



Braun et al.

Pion

Spin Asymmetries

Transverse spin density

$\Downarrow$

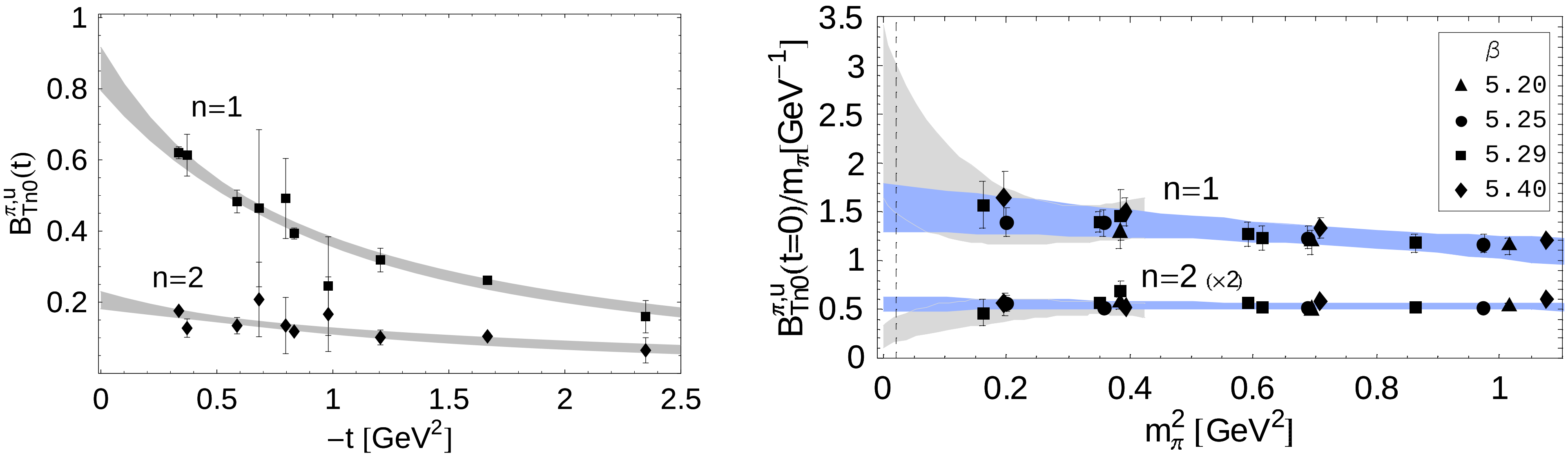
$\lambda_\perp$ quark spin

$$\langle p_+|\bar{q}(\mathbf{b}_\perp)\left[\gamma_+ - \boldsymbol{\lambda}_{\perp i}\,\boldsymbol{\sigma}_{+j}\,\gamma_5\right]q(\mathbf{b}_\perp)|p_+\rangle = \left\{A_1^q(\mathbf{b}_\perp^2) - \frac{1}{m_\pi}\epsilon_{ij}b_{\perp j}\boldsymbol{\lambda}_{\perp i}\,\bar{B}_1^{Tq}(\mathbf{b}_\perp^2)'\right\}$$

$\Uparrow$ Monopole

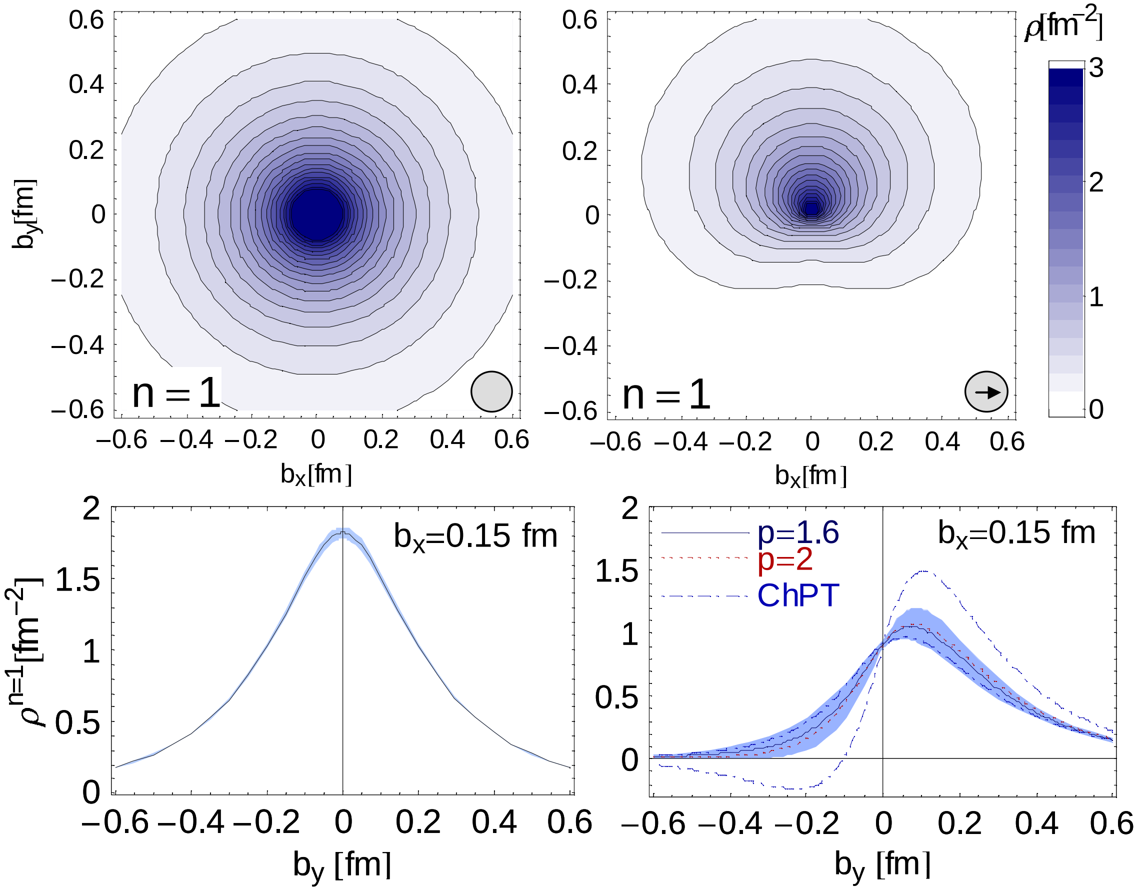
$\Uparrow$ Dipole

Hägler et al.



$B^{\pi,u}_{Tn0}(t)$
n=1
n=2
$-t$ [GeV^2]
$B^{\pi,u}_{Tn0}(t=0)/m_\pi$[GeV^{-1}]
n=1
n=2 (×2)
m_π^2 [GeV^2]
β
5.20
5.25
5.29
5.40

$p-$pole fit



Boer-Mulders effect

ChPT: Diehl, Manashov & Schäfer

Distribution Amplitude

Expansion

$$\phi_\pi(\xi,\mu^2) = \frac{3}{4}(1-\xi^2)\,\left(1 + \sum_{n=1}^{\infty} a_n(\mu_0)\, C_n^{3/2}(\xi)\, L^{\gamma n/2\beta_0}\right)$$

$$L = \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}$$

$$\gamma_n = \frac{32}{3}\left(\sum_{k=1}^{n+1}\frac{1}{k} - \frac{3}{4} - \frac{1}{2(n+1)(n+2)}\right)$$

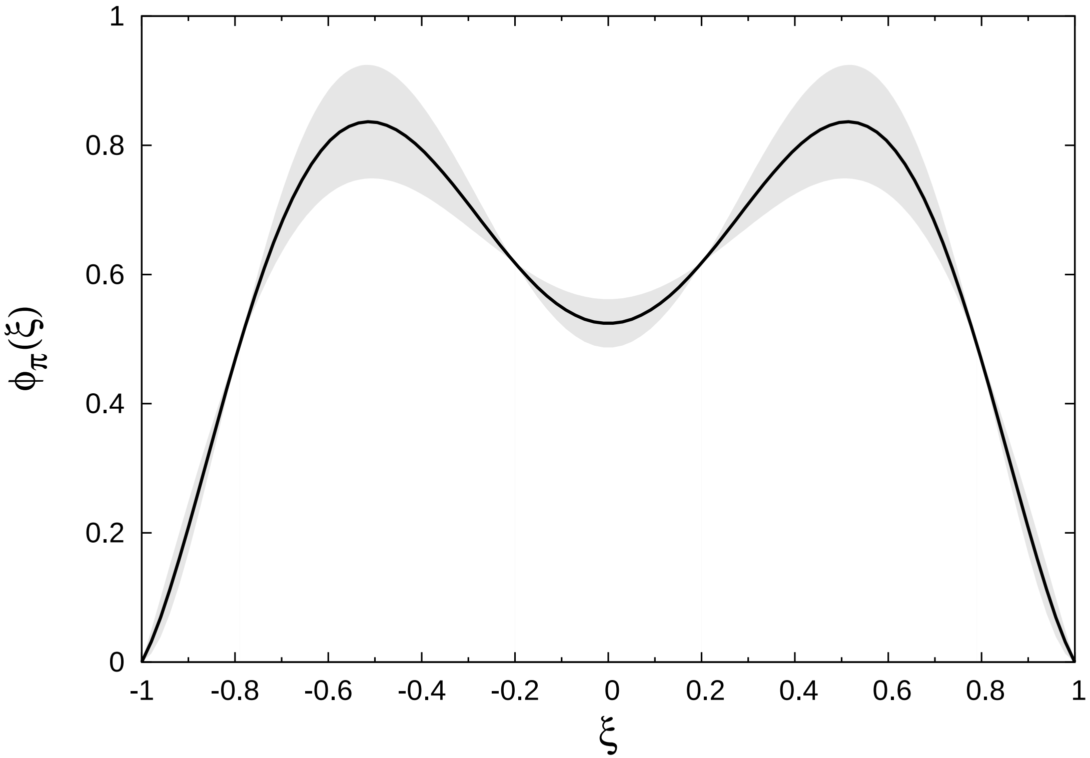
$$a_1 = \frac{5}{3}\,\langle\xi\rangle$$

$$a_2 = \frac{7}{12}\left(5\langle\xi^2\rangle - 1\right)$$

$$\vdots$$

$$\langle\xi^n\rangle = \int_{-1}^{1} d\xi\,\xi^n \phi_\pi(\xi,\mu^2)$$

$\phi_\pi(\xi, \mu^2)$



$\mu^2 = 4\ \text{GeV}^2$

Conclusions & Outlook

- Simulations at the physical pion mass with Wilson-type fermions are feasible now

- Extrapolations to infinite volume are largely under control

- Simulations with $N_f = 2 + 1$ SLiNC fermions are under way

- Challenge: Evaluation of disconnected diagrams

- To exploit the full potential of lattice calculations, interactions with Experiment and Hadron Phenomenology are very important

- Improvement of algorithms
- Increase of computing power

FS corrections surprisingly well described by ChPT

Field of active research at DESY, JLab, BNL, CERN, $\cdots$