From glueballs to large N: strong coupling physics from the lattice

Michael Teper (Oxford)

Sapporo '09

Pioneering lattice calculations of glueball masses with Kenzo Ishikawa – how do they compare with modern results?

Is $N = \infty$ physically relevant: i.e. is large-N confining and is $N = 3$ close to $N = \infty$?

What string theory describes confining flux tubes in $D = 3$ and $D = 4$?

Strong coupling physics from the lattice – beyond QCD

Lattice – Preamble

Wilson 1974 ; Creutz 1979-80

- Euclidean $R^4 \longrightarrow$ hypercubic lattice on T^4 : finite problem

- comparing colour:

continuum infinitesimal: $x_\mu \bullet - \bullet\, x_\mu + \hat{\mu}\delta x$: $A_\mu(x) \in$ SU(N) Lie Algebra

$\longrightarrow$

continuum finite: $x_\mu \bullet - - - \bullet\, x'_\mu$: $P\left\{e^{\int_x^{x'} A.dx}\right\} \in$ SU(N) group

$\xrightarrow{x_\mu = a n_\mu}$

finite on lattice: $a n_\mu \bullet - - - \bullet\, a n_\mu + a\hat{\mu}$: $U_\mu(n) \in$ SU(N) group

i.e. SU(N) matrices U_l on each link l

- gauge transformation: $U_\mu(n) \to g(n) U_\mu(n) g^\dagger(n+\hat{\mu})$

$\longrightarrow$

$\mathrm{Tr} \prod_{l \in \partial c} U_l$ gauge invariant for any closed curve c

$\longrightarrow$

so $\quad Z = \int \prod_l dU_l e^{-\beta S} \quad$ where $\quad S = \sum_p \{1 - \frac{1}{N} \mathrm{ReTr} u_p\}$

and u_p is product links around the plaquette p is a suitable, although not unique, SU(N) lattice gauge theory

- symmetries ensure that:

$$\int \prod_l dU_l e^{-\beta S} \quad \xrightarrow{a \to 0} \quad \int DA e^{-\frac{4}{g^2} \int d^4x Tr F_{\mu\nu} F_{\mu\nu}} \text{ with } \beta = \frac{2N}{g^2(a)} \xrightarrow{a \to 0} \infty$$

and we vary the parameter β in order to vary the lattice spacing a

- Monte Carlo: $\quad Z^{-1} \int \prod_l dU_l \Phi(U) e^{-\beta S} = \frac{1}{n} \sum_{I=1}^{n} \Phi(U^I) + O(\frac{1}{\sqrt{n}})$

Calculating masses : Wilson, Coseners House, March 1981

• write the Euclidean correlator of an operator $\phi(t)$:

$$\langle\phi^\dagger(t=an_t)\phi(0)\rangle = \langle\phi^\dagger e^{-Han_t}\phi\rangle = \sum_i |c_i|^2 e^{-aE_i n_t} \stackrel{t\to\infty}{=} |c|^2 e^{-man_t}$$

where am is lightest mass (in lattice units) with quantum numbers of ϕ. In particular, take $\vec{p}=0$, colour singlet, and some particular J^{PC}.

• in a numerical calculation, with finite errors, we need to be able to calculate am at small t before the 'signal' has become too small i.e. when $|c|^2$ is large and ϕ is a good wavefunctional for the desired ground state

• so generalise this to a variational calculation over a vector space V_ϕ spanned by some convenient operators $\{\phi_i; i = 1, ..., n\}$ of the desired quantum numbers:

$$\langle {\psi_0}^\dagger(t_0)\psi_0(0)\rangle = \max_{\phi \in V_\phi} \langle \phi^\dagger(t_0)\phi(0)\rangle = \max_{\phi \in V_\phi} \langle \phi^\dagger e^{-Ht_0}\phi\rangle$$

where t_0 is some convenient value of t. Then ψ_0 is our best variational estimate for the true eigenfunctional of the ground state (with these quantum numbers). We can now use $\langle {\psi_0}^\dagger(t)\psi_0(0)\rangle$ to obtain our best estimate of the ground state mass.

• generalise this in an obvious way to calculating excited state energies

$\longrightarrow$ the first glueball lattice calculations ....

• The glueball mass spectrum in QCD: First results of a lattice Monte Carlo calculation.
K. Ishikawa, M. Teper (DESY), G. Schierholz (Hamburg U.)
DESY 81/089, (Received 18 January 1982) Phys.Lett.B110:399,1982
TOPCITE = 50+

• Monte Carlo estimates of the SU(2) mass gap.
B. Berg, A. Billoire (CERN), C. Rebbi (Brookhaven)
BNL-30826, Dec 1981. (Received 8 February 1982) Annals Phys.142:185-215,1982, Addendum-ibid.146:470-472,1983.
TOPCITE = 50+

• On the masses of the glueballs in pure SU(2) lattice gauge theory
M. Falcioni, E. Marinari, M. L. Paciello, G. Parisi, F. Rapuano, B. Taglienti and Zhang Yi-Cheng (INFN, Roma)
ROME-277-1981, Dec 1981. (Received 6 January 1982) Phys.Lett.B110:295,1982.
TOPCITE = 50+

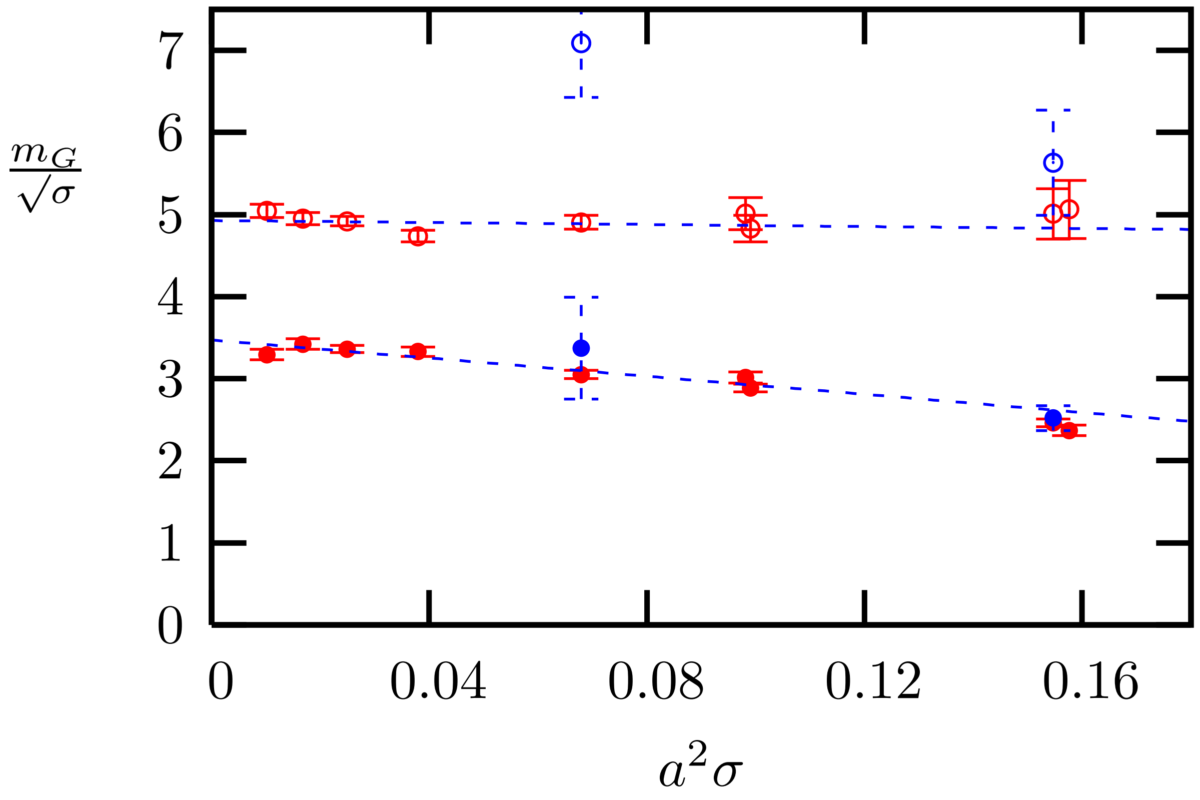
How do these pioneering calculations compare to recent calculations of the continuum limit?

• to obtain the continuum limit from masses that are in lattice units and are distorted by the finite lattice cutoff, take dimensionless mass ratios and extrapolate with an $O(a^2)$ correction, Symanzik early-80's , e.g.

$$\frac{am(a)}{a\sqrt{\sigma(a)}} = \frac{m(a)}{\sqrt{\sigma(a)}} = \frac{m(0)}{\sqrt{\sigma(0)}} + c_0 a^2 \sigma + O(a^4)$$

where we choose to use the square root of the string tension σ as one of the masses (here c_0 is a power series in the bare coupling, but this logarithmic variation with a can usually be ignored)

SU(3) : K. Ishikawa et al, PL 116B (1982) 429 ; B.Lucini et al: hep-lat/0404008



$O(a^2)$ extrapolations to $a = 0$:

(●) $\frac{m_{0^{++}}}{\sqrt{\sigma}} = 3.47(4) - 5.52(75)a^2\sigma$; (○) $\frac{m_{2^{++}}}{\sqrt{\sigma}} = 4.93(5) - 0.61(1.36)a^2\sigma$

$\longrightarrow$ the only glueball mass with small errors is exactly on the modern calculations!

Note:
in 1982 we used $a\sqrt{\sigma}$ values from other people's papers, and these values were in fact off by about 40% so that in fact:

$$\frac{m_{0^{++}}}{\sqrt{\sigma}} \stackrel{\sigma:1982}{\simeq} 1.8 \stackrel{\sigma:2004}{\longrightarrow} 2.5$$

and, as we see, lattice corrections are substantial, so that:

$$\frac{m_{0^{++}}}{\sqrt{\sigma}} \stackrel{a\sim 0.4/\sqrt{\sigma}}{\simeq} 2.5 \stackrel{a\to 0}{\longrightarrow} 3.5$$

$\longrightarrow$

$$m_{0^{++}} \simeq 3.5\sqrt{\sigma} \simeq 1.6\text{GeV}$$

which fits in with the three observed $J^{PC} = 0^{++}$ flavour 'singlet' states $f_0(1350), f_0(1500), f_0(1700)$ coming from mixing of nearby $u\overline{u} + d\overline{d}$, $s\overline{s}$ and glueball states

our first papers developed all the main technical ingredients of modern calculations, except for one major problem :
K. Ishikawa etal, Z. Phys. C 16 (1982) 69

if we use simple Wilson loops of various sizes, then

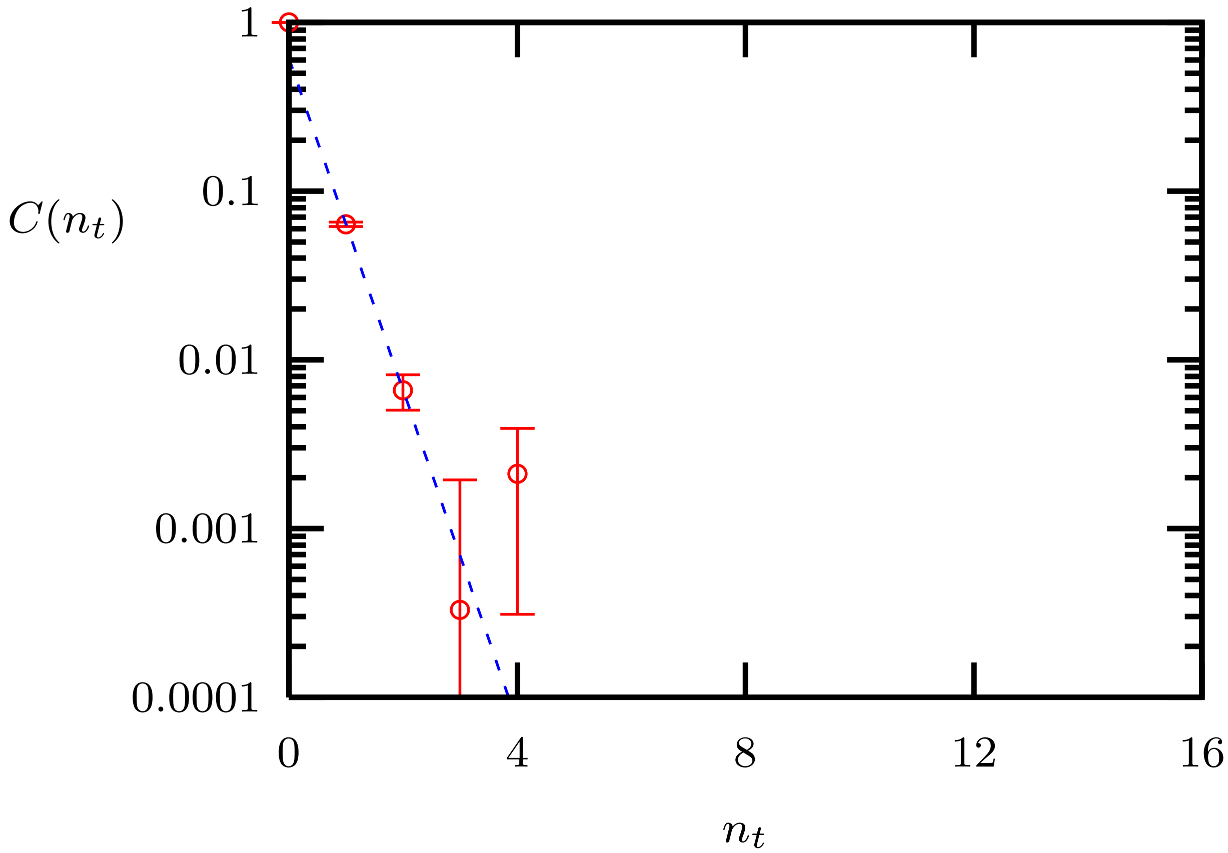
$$\text{the overlap onto ground state} \twoheadrightarrow 0 \text{ as } a \longrightarrow 0$$

and the 'signal' is lost in the 'noise' before we can identify the mass of the ground state

e.g. take the smallest a in our plot i.e.
SU(3), 32^4, $a \simeq 0.046$ 'fm'
and use the simple plaquette for the glueball operator

$$\longrightarrow$$

using the simple plaquette for the glueball operator:



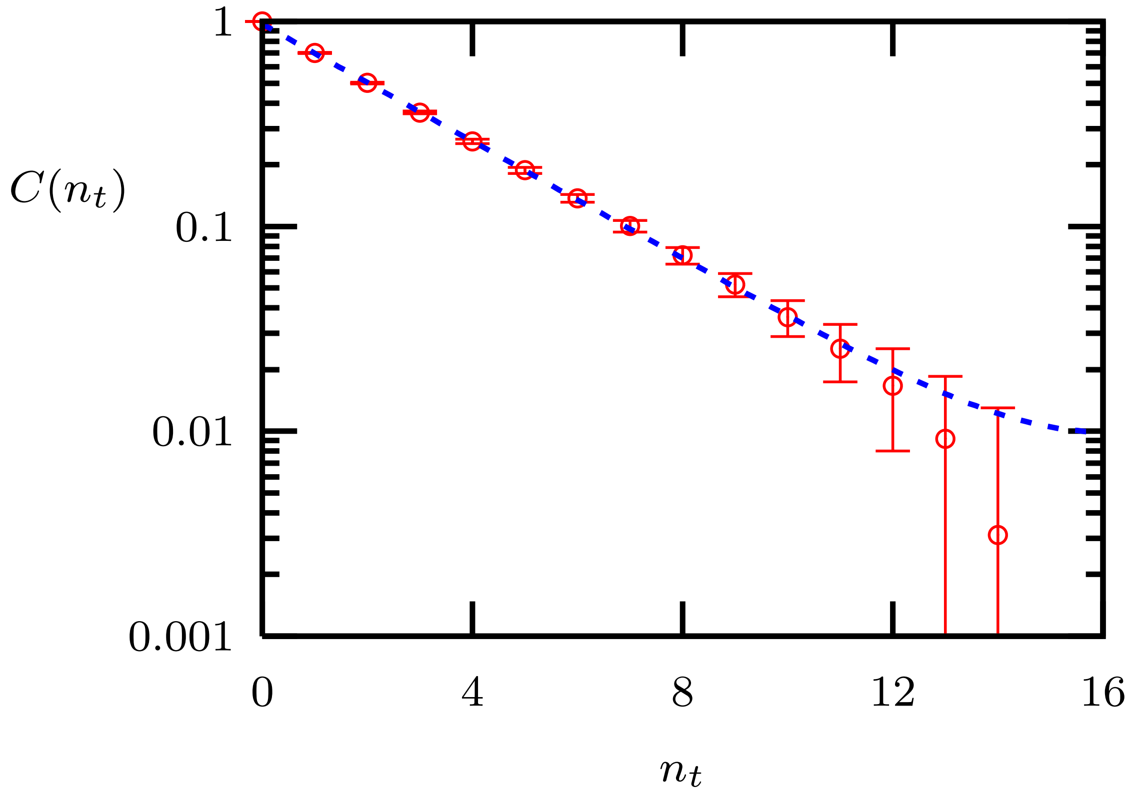
$C(t) \propto e^{-amn_t}$ at larger $t = an_t$?!

this necessitated the one qualitative improvement to our original calculations (apart from more powerful computing!)

produce operators that are 'smooth' on physical length scales by the iterative 'smearing' and 'blocking' of the lattice gauge fields used in constructing appropriate Wilson loops

$\longrightarrow$

best blocked/smeared glueball operator



$C(t = an_t) \overset{t\uparrow}{\simeq} |c|^2 e^{-man_t} \quad \Rightarrow \quad$ fit : $am_{0^{++}} = 0.330(7)$

Our glueball papers in the early 1980's:

The glueball mass spectrum in QCD: first results of a lattice Monte Carlo calculation.
K. Ishikawa, G. Schierholz, M. Teper
Phys.Lett.B110:399,1982.
Cited 88 times

SU(3) lattice Monte Carlo calculation of the glueball mass spectrum.
K. Ishikawa, G. Schierholz, M. Teper
Phys.Lett.B116:429,1982.
Cited 69 times

Renormalization group behavior of 0+ and 2+ glueball masses in SU(2) lattice gauge theory.
K. Ishikawa, G. Schierholz, M. Teper
Z.Phys.C16:69,1982.
Cited 11 times

Prediction of low lying oddballs in lattice QCD.
K. Ishikawa, A. Sato, G. Schierholz, M. Teper
Phys.Lett.B120:387,1983.
Cited 26 times

On investigating the structure of hadrons: lattice Monte Carlo measurements of color magnetic and electric fields and the topological charge density inside glueballs.
K. Ishikawa (CCNY), G. Schierholz, H. Schneider, M. Teper (LAPP)
Nucl.Phys.B227:221,1983.
Cited 17 times

Calculation of the glueball mass spectrum of SU(2) and SU(3) non-Abelian lattice gauge theories 1. Introduction and SU(2).
K. Ishikawa, G. Schierholz, M. Teper
Z.Phys.C19:327,1983.
Cited 40 times

On the topological structure of the vacuum in SU(2) and SU(3) lattice gauge theories.
K. Ishikawa, G. Schierholz, H. Schneider, M. Teper
Phys.Lett.B128:309,1983.
Cited 39 times

Calculation of the glueball mass spectrum of SU(2) and SU(3) non-Abelian lattice gauge theories 2. SU(3).
K. Ishikawa, A. Sato, G. Schierholz, M. Teper
Z.Phys.C21:167,1983.
Cited 61 times

Full QCD:

- PACS-CS Collaboration:

Y. Kuramashi, Plenary Talk at Lattice 2008, arXiv:0811.2630

S. Aoki et al., arXiv:0807.1661

- S. Durr et al., Science 322 (2008) 1224

Large N

't Hooft 1974

- Consider SU(N) as a power series in $1/N$ around SU(∞)

- If:

SU(N) is confining $\forall N$ and SU(3) is close to SU(∞)
then there is a lot of mysterious strong-interaction physics whose origins one can understand by taking $N \to \infty$ while keeping $g^2 N$ fixed
't Hooft, Coleman, Witten, Veneziano, Dahen, Manohar...

- In a confining phase at $N = \infty$, there are no interactions between colour singlet states, no decays, no mixing – but still the theory has proved too difficult to solve

• Since 1997 Maldacena new hope of a solution at $N = \infty$ has been provided by the strong-weak coupling gauge-gravity dualities and this has provided new motivation for numerical calculations at large N

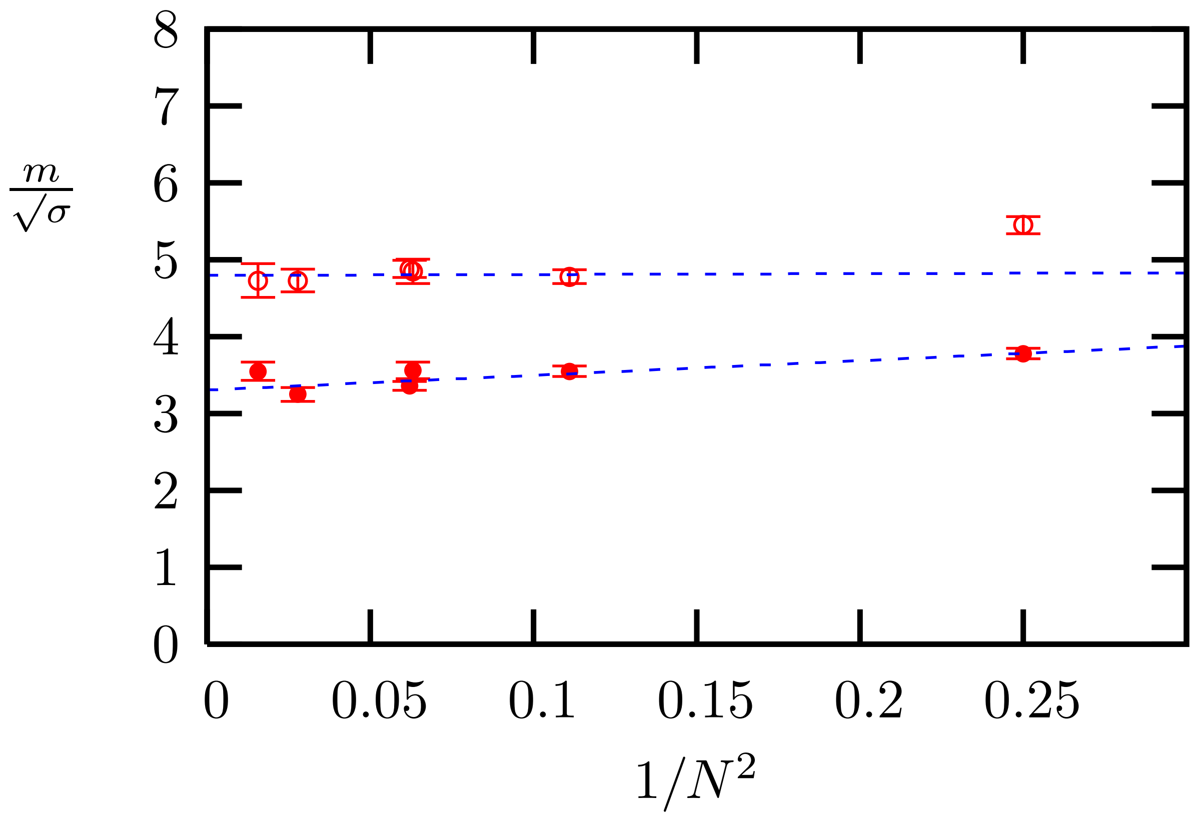
• A simple and effective strategy is to repeat the calculations for larger N and compare the results, i.e. SU(2), SU(3), SU(4), SU(5), SU(6), ...

• Since the leading correction in a theory with just adjoint fields is expected to be $O(1/N^2)$, going to say $N = 8$ should be more than enough, and use

$$\frac{m(N)}{\sqrt{\sigma(N)}} = \frac{m(\infty)}{\sqrt{\sigma(\infty)}} + \frac{c}{N^2} + O\left(\frac{1}{N^4}\right)$$

Glueball mass spectrum: large-N limit

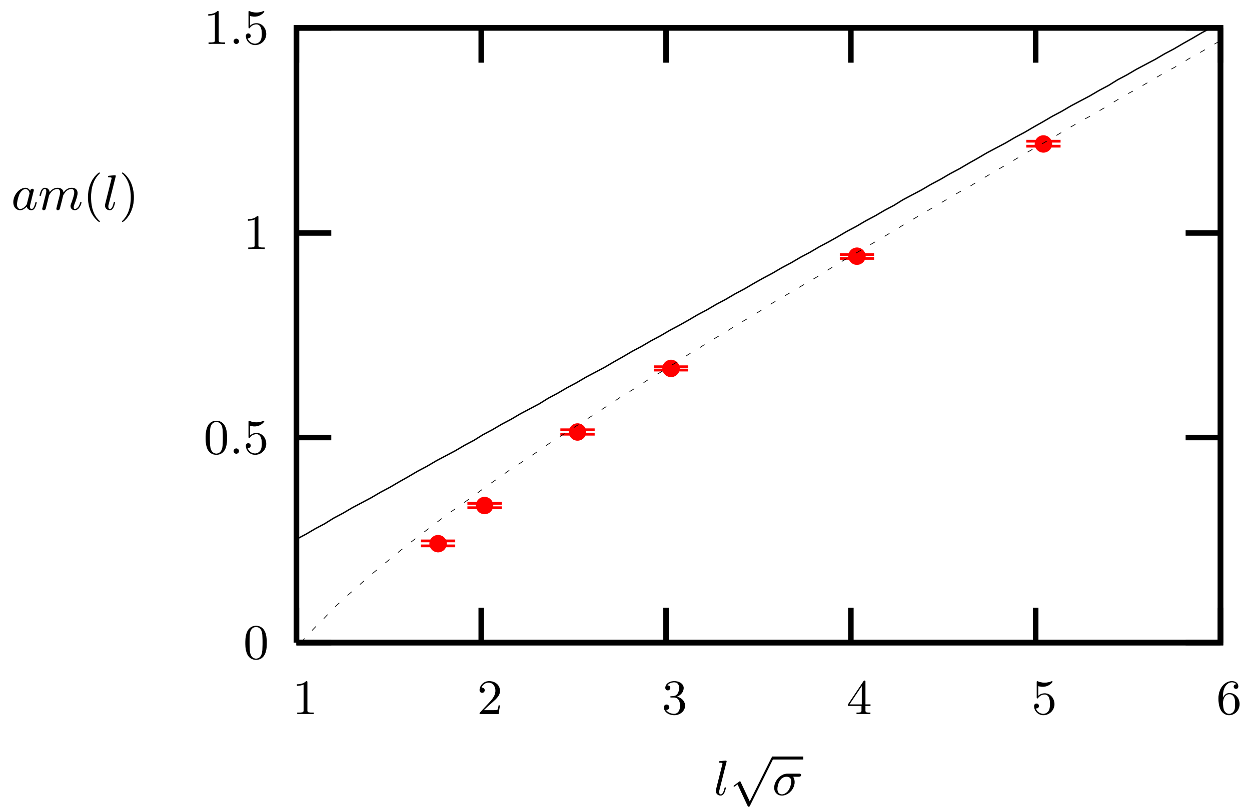
B.Lucini, M.Teper, U.Wenger: hep-lat/0404008



(•) 0^{++}; (∘) 2^{++} $\longrightarrow$ SU(3) is 'close to' SU(∞) for many quantities

SU(6) : energy of flux loop closed around a spatial torus

H. Meyer, M. Teper: hep-lat/0411039

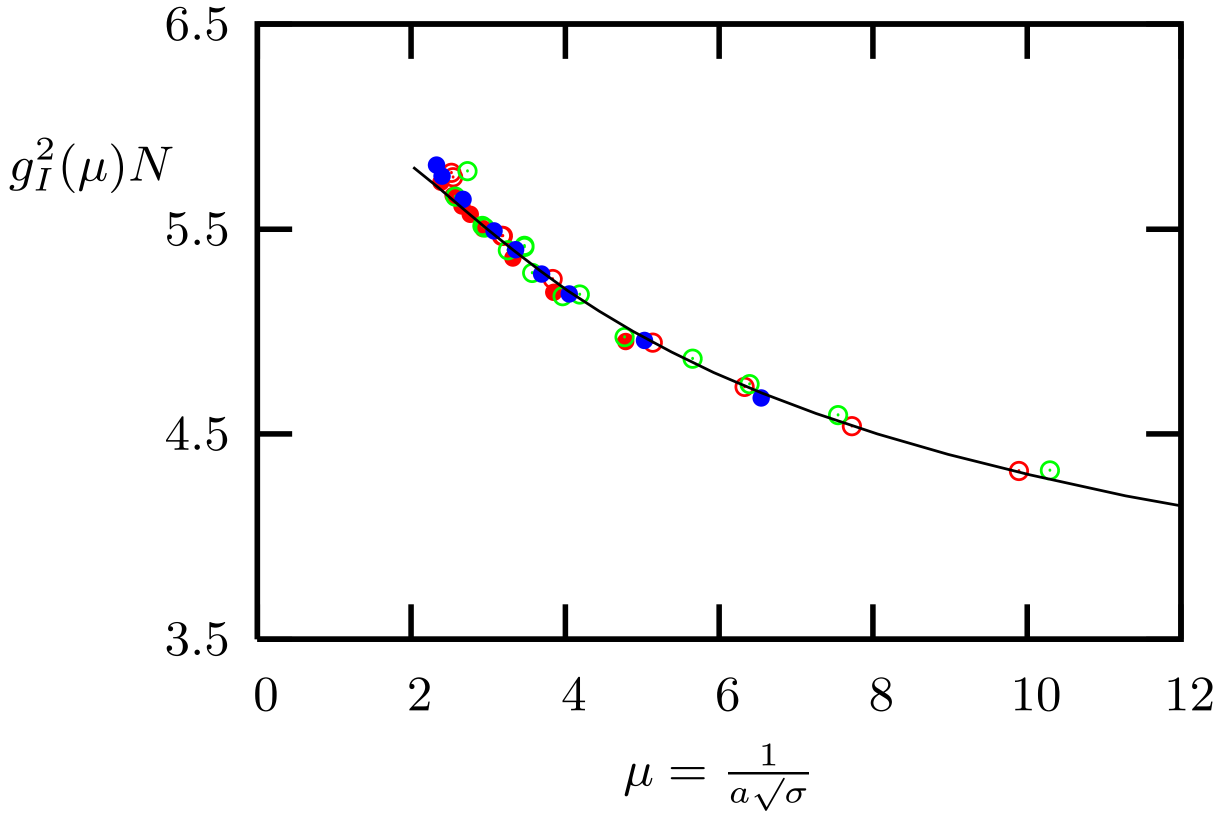


$\longrightarrow$ linear confinement: $am(l) \simeq \sigma l - \frac{\pi}{3l}$

g^2N fixed as $N \to \infty$?

MT, Lat 08 , arXiv:0812.0085

bare coupling (Parisi): $g_I^2(a) = \frac{g^2}{u_p} = \frac{2N}{\beta}\frac{1}{u_p}$



SU(2) ○ ; SU(3) ○; SU(4) ● ; SU(6) ○ ; SU(8) ●

• from a careful analysis we can obtain:

C. Allton, M. Teper, A. Trivini, arXiv:0803.1092

$$\frac{\Lambda_{\overline{MS}}}{\sqrt{\sigma}} = 0.503(2)(40) + \frac{0.33(3)(3)}{N^2}$$

• to go beyond a bare coupling to a genuine continuum running coupling, one can use e.g. the SF coupling scheme, giving :

B. Lucini, G. Moraitis, arXiv:0805.2913, 0710.1533

$$\frac{\Lambda_{\overline{MS}}}{\sqrt{\sigma}} = 0.528(40) + \frac{0.18(36)}{N^2}$$

QUESTION: does the SF coupling acquire non-perturbative jumps at the Narayanan-Neuberger $N = \infty$ phase transitions?

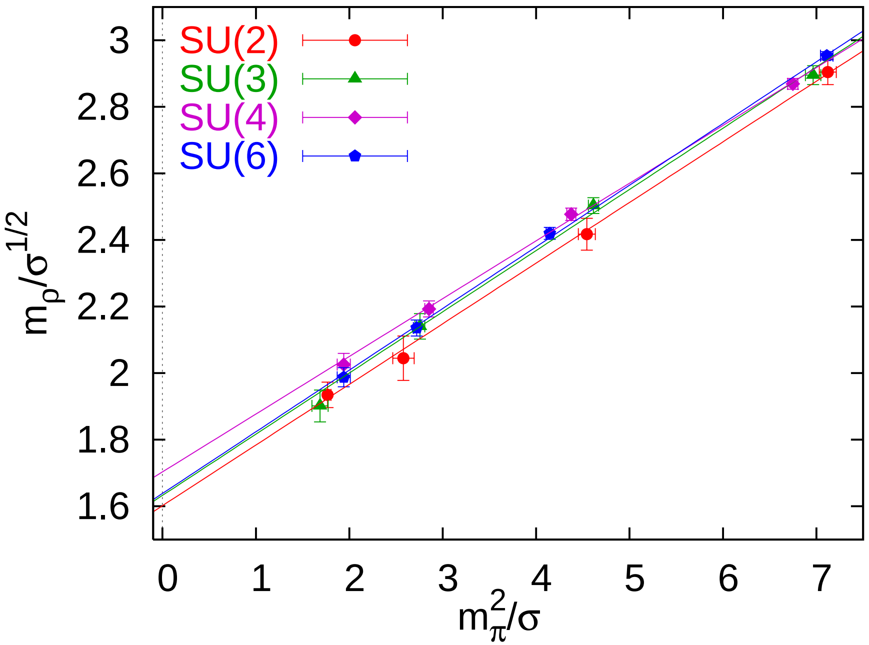
QCD at $N = \infty$

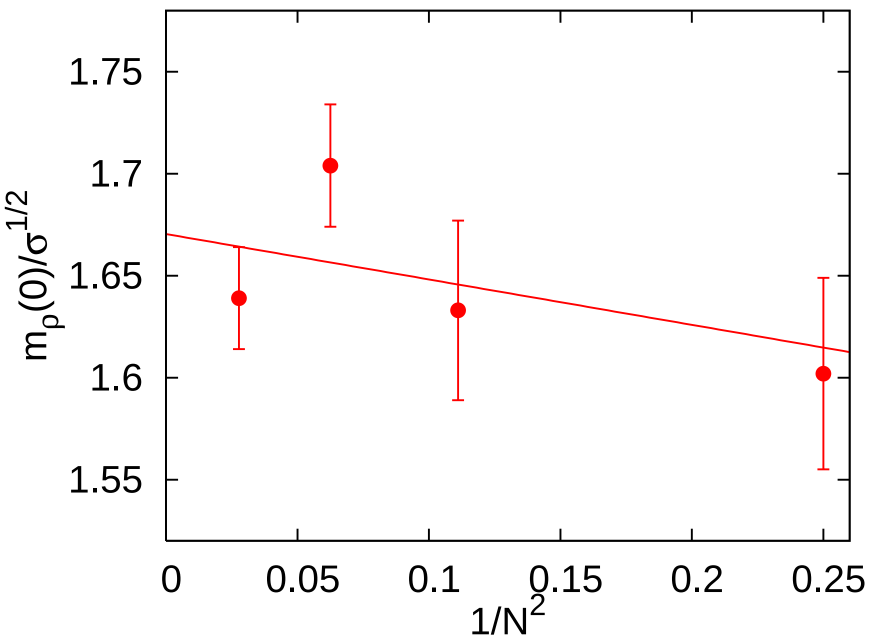
Note : $\quad$ QCD $\stackrel{N=\infty}{\equiv}$ quenched QCD

- L. Del Debbio, B. Lucini, A. Patella and C. Pica: arXiv:0712:3036.
- G. Bali and F. Bursa, arXiv:0806:2278; arXiv:0708:3427.

Some questions:
Scalar mesons as $N \to \infty$: do the $\leq$ 1GeV states disappear?
The scalar nonet and the place of lightest scalar glueball?
Flavour singlet tensor and pseudoscalar mesons and glueballs?
Excited states stable $\longrightarrow$ Regge trajectories?
Excited states stable $\longrightarrow$ clean meson excitation spectrum.
SU($2n_f$) baryon (Dashen-Manohar) symmetry as $N \to 3$.

G. Bali and F. Bursa, arXiv:0806:2278





m_ρ versus m_π (left); m_ρ for $m_q = 0$ versus $1/N^2$ (right).

Del Debbio et al: $\lim_{N\to\infty} \frac{m_\rho}{\sqrt{\sigma}} = 1.627(10) \quad ; \qquad a\sqrt{\sigma} = 0.335$

$+$

Bali and Bursa: $\lim_{N\to\infty} \frac{m_\rho}{\sqrt{\sigma}} = 1.688(25) \quad ; \qquad a\sqrt{\sigma} = 0.209$

$\longrightarrow$

$$\lim_{N\to\infty, a\to 0} \frac{m_\rho}{\sqrt{\sigma}} = 1.79(5)$$

versus, in the real world :

$$\frac{m_\rho}{\sqrt{\sigma}} \simeq \frac{770\mathrm{MeV}}{440\mathrm{MeV}} \simeq 1.75$$

$\longrightarrow$

$N = 3$ is 'close to' $N = \infty$ for full QCD ...

Flux tubes as strings

$\longleftrightarrow$

Confining flux tubes in SU(N) gauge theories and their effective string theory description

Athenodorou, Bringoltz, MT: arXiv:0812.0334; 0802.1490; 0709.0693

- Veneziano amplitude
- 't Hooft large-N – genus diagram expansion
- Polyakov action

and much more recently

- Maldacena ... AdS/CFT/QCD ...

This has been an area of active and continuing research on the lattice since 1980 ...

focus on spectrum of flux tubes that
are closed around a spatial torus of length l :

• flux localised in 'tubes' $\forall l \geq l_c = 1/T_c$

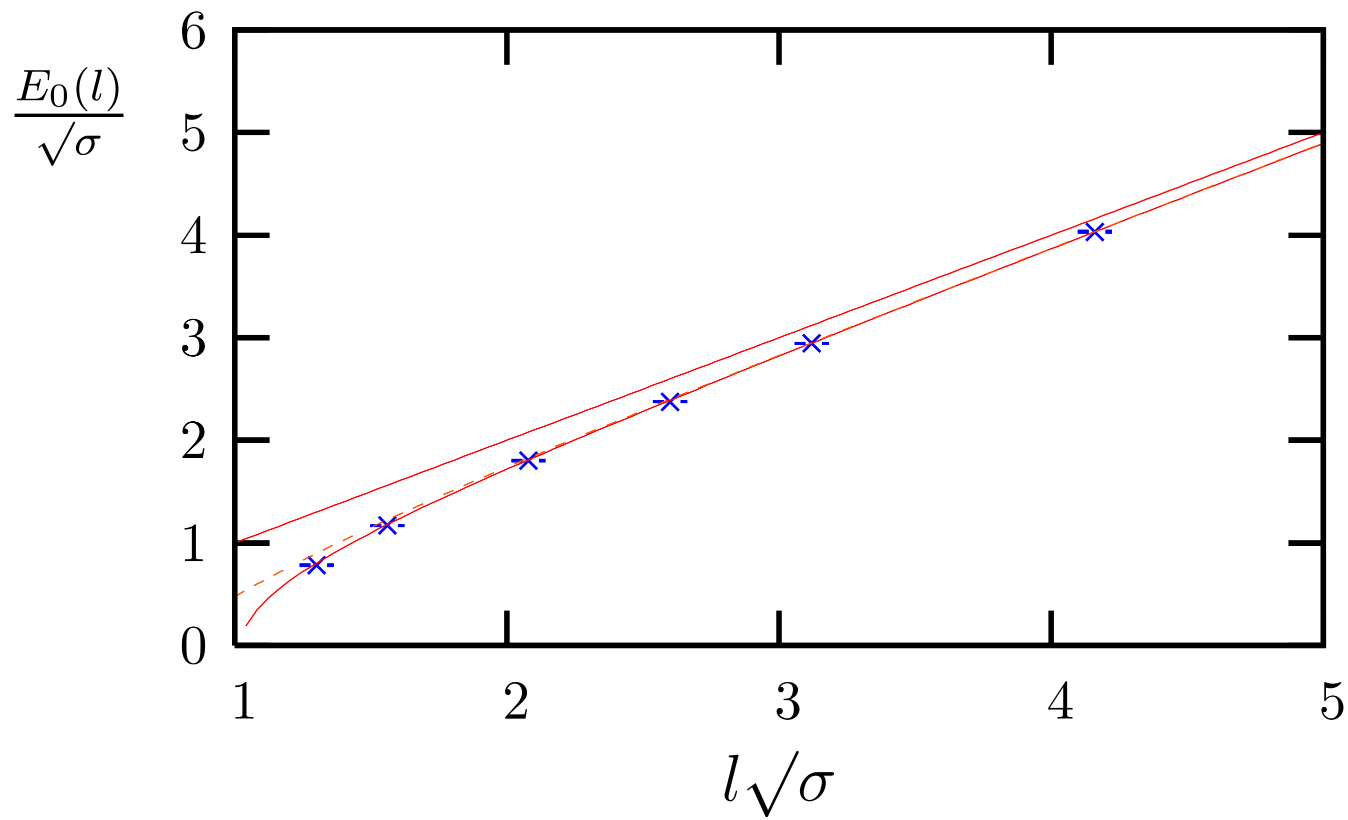
• at $l = l_c$ there is a phase transition: first order for $N \geq 3$ in $D = 4$ and for $N \geq 4$ in $D = 3$

• so may have a simple string description of the closed string spectrum for all possible lengths (large N)

• most plausible at $N \to \infty$ where complications such as mixing, e.g string $\to$ string + glueball, go away

the static potential $V(r)$ describes the transition in r between UV (Coulomb potential) and IF (flux tubes) physics; potentially of great interest as $N \to \infty$.

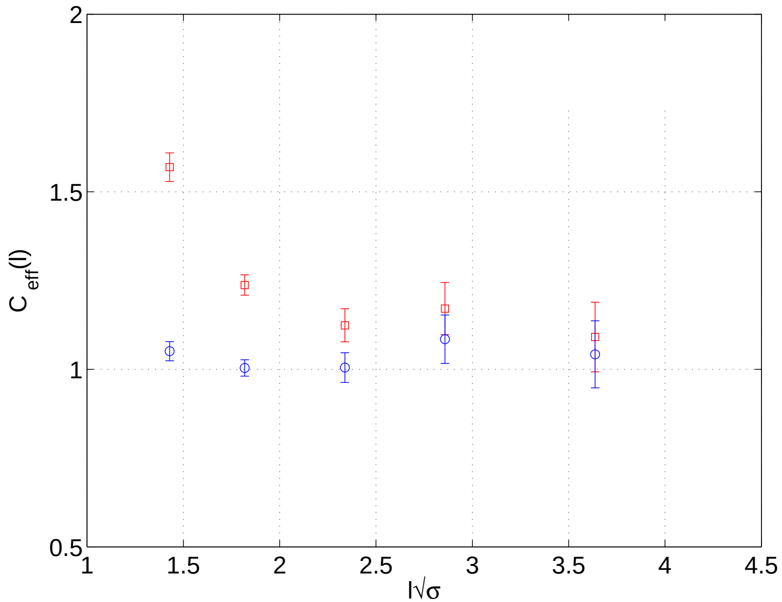
Closed flux tube in D=2+1 and SU(5)



Luscher: (...) $E_0(l) = \sigma l - \frac{\pi}{6l}$

Nambu-Goto: (−) $E_0(l) = \sigma l \left(1 - \frac{\pi}{3\sigma l^2}\right)^{\frac{1}{2}}$

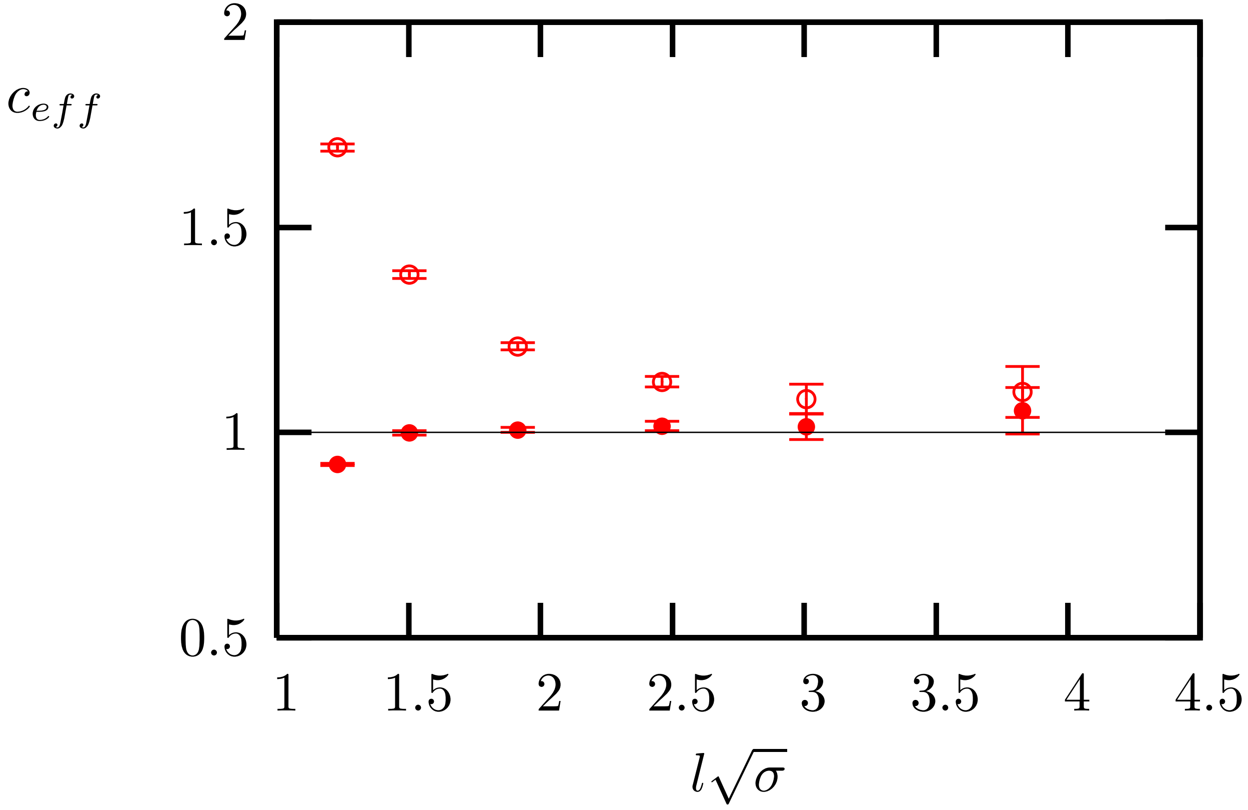
How well do Luscher and Nambu-Goto work here?



□ : c_{eff} from Luscher: $E_0(l) = \sigma l - \frac{c_{eff}\pi}{6l}$

○ : c_{eff} from Nambu-Goto: $E_0(l) = \sigma l \left(1 - \frac{c_{eff}\pi}{3\sigma l^2}\right)^{\frac{1}{2}}$

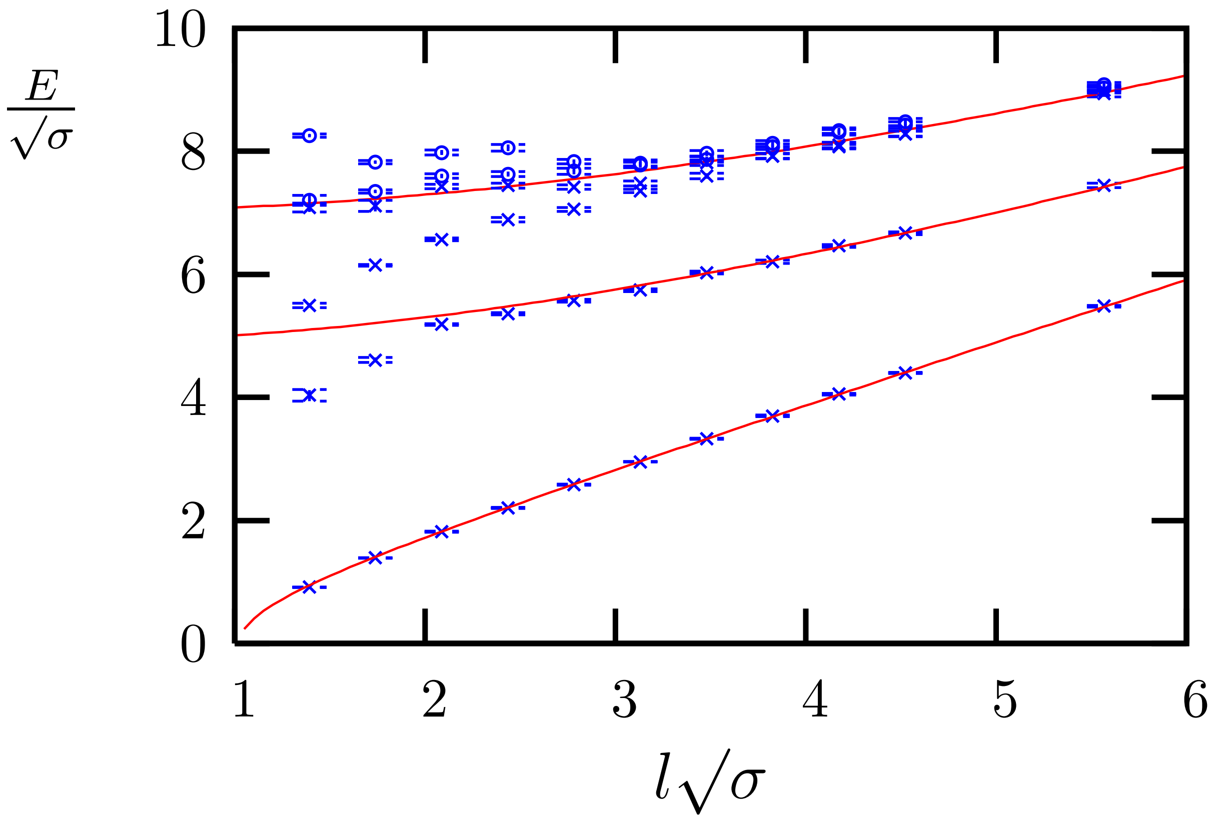
more accurate for SU(2) : $l_c\sqrt{\sigma} \simeq 0.94$



○ : c_{eff} from Luscher

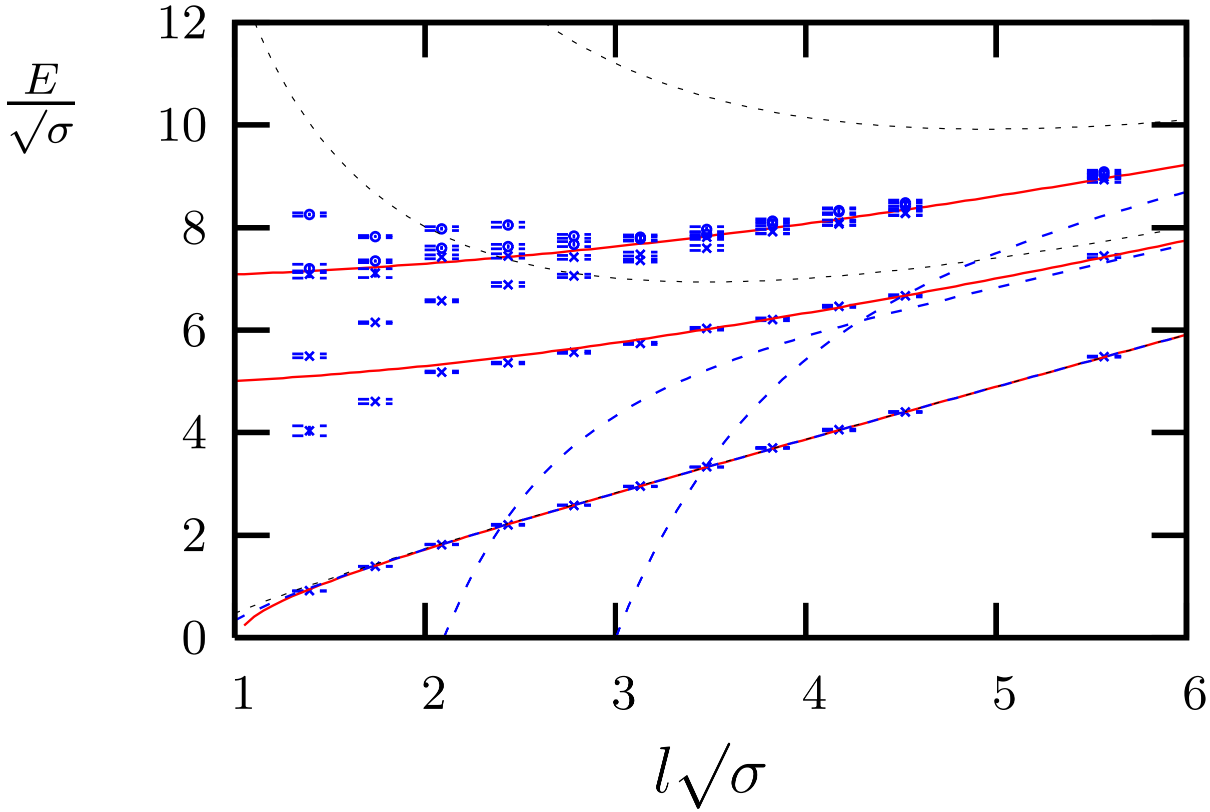
● : c_{eff} from Nambu-Goto

SU(3) : closed string spectrum for $a\sqrt{\sigma} = 0.17395(7)$; $l_c\sqrt{\sigma} \simeq 1.0$



- Nambu-Goto: (σ from ground state); $\times$: +ve parity; $\circ$: -ve parity

closed string spectrum - Nambu-Goto vs Luscher



— Nambu-Goto : $E_n = \sigma l \sqrt{1 + \frac{8\pi}{\sigma l^2}\left(n - \frac{1}{24}\right)}$

...... Luscher 1980,2004: $E_n = \sigma l + \frac{4\pi}{l}\left(n - \frac{D-2}{24}\right) - \frac{8\pi^2}{\sigma\, l^3}\left(n - \frac{1}{24}\right)^2$

$\longrightarrow$

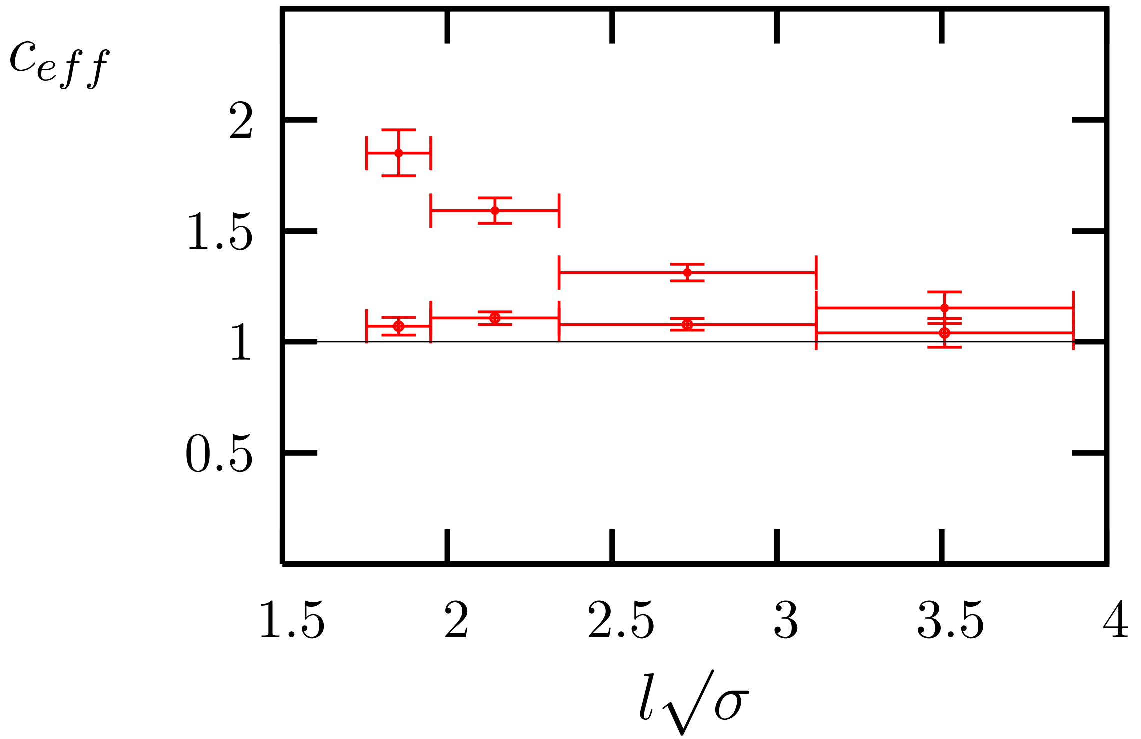
- the right starting point for understanding confining flux tubes is the free Nambu-Goto string theory rather than some large-l truncation thereof

- we have seen that even very short flux tubes – really ‘blobs’ not ‘tubes’ – know that they are really strings: this is not natural in a generic ‘Nielsen-Olesen vortex’ picture, but it is what happens in gauge-gravity duals.

- what are the inter-phonon interactions?
where are the massive (e.g. breathing) modes?

What about D=3+1? [SU(3) ; $\beta = 6.0625$; $l_c\sqrt{\sigma} \sim 1.6$]

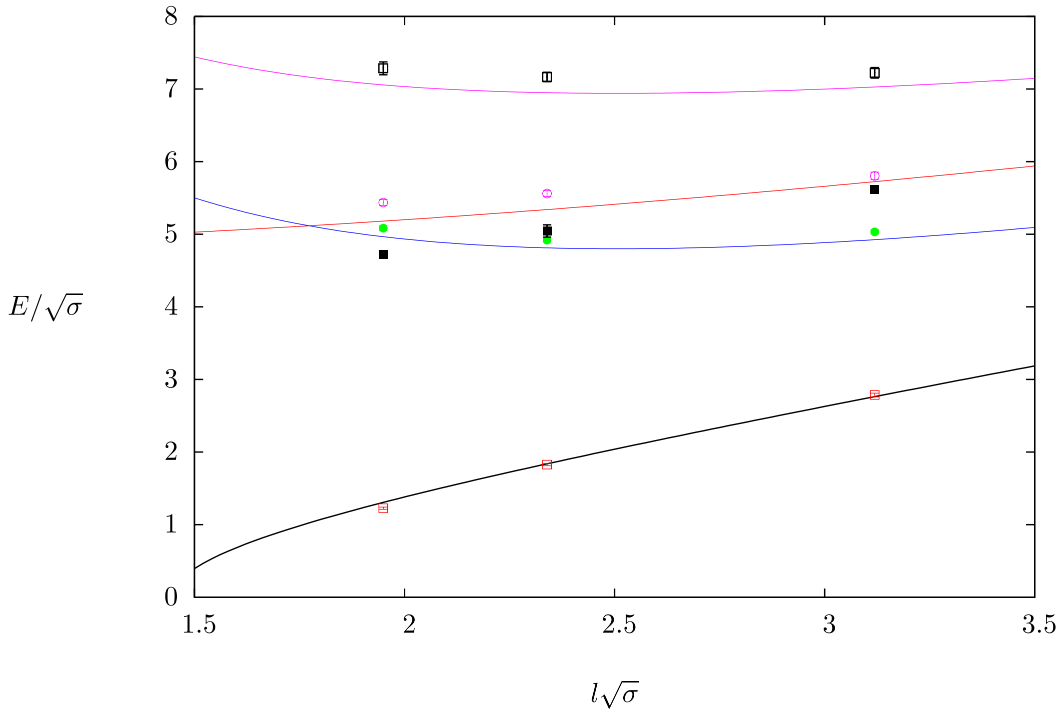
○ Nambu-Goto; ● Luscher



$20 \times 20^2 \times 16$, 4.5M sweeps ; $16 \times 16^2 \times 16$, 6.0M sweeps ; $12 \times 16^2 \times 24$, 5.0M sweeps
$10 \times 20^2 \times 36$, 2.0M sweeps ; $9 \times 32^2 \times 48$, 0.6M sweeps

spectrum of excited states (preliminary) ...

lines Nambu-Goto: only parameter is σ



●$q = 0$ ground state: $J = 0$ ●$q = 1$ ground state: $J = 1$ ●$q = 0$ excited state: $J = 0$

●$q = 0$ excited state: $J = 2$ ○$q = 1$ excited state: $J = 1$

What is today's big lattice problem?

$\leftrightarrow$

What is *today's* big physics problem?

$\leftrightarrow$

LHC and BSM

$\longrightarrow$

strong coupling BSM physics
e.g. near conformal ETC or SUSY