# Generalized Gauge Theory : An Overview

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with
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# I. Introduction

**Formulation towards Unified Model on Lattice**

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**Generalized Gauge Theory**

◇ **Suggestive results**

⋆ **Two-dimensional Quantum Gravity**

$\longleftarrow$ dynamical triangulation of random surface

— David, Kazakov-Kostov-Migdal, Ambjørn-Durhuus-Fröhlich

— Knizhnik-Polyakov-Zamolodchikov, David, Distler-Kawai (Liouville theory)

$$Z = \int \mathcal{D} g_{\mu\nu} e^{-S(g)} \sim \sum_{T:\ \text{triangulations}} e^{-S(T)}$$

- $g \longrightarrow T$: The curvature is concentrated **on the sites** of triangles.

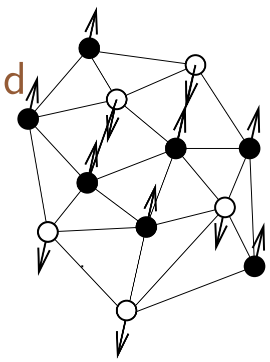
◇ **Matter on the lattice**

$\longleftarrow$ ·gravitational background: dynamical triangulation

·matter: some degrees of freedom **on a simplex** of the simplicial manifold

(e.g. Ising model $\pm 1$ values **on the sites** (0-simplex))

— Bershadsky-Migdal, Boulatov-Kazakov (matrix model)



## ⋆ Dirac-Kähler fermion

:curved space version of Kogut-Susskind lattice fermions (with **flavor**)

$\sim$ staggered fermion ($\longleftarrow$ Kawamoto-Smit trans.)

Laplacian can be expressed in terms of $d$ and its dual $\delta(\sim *d*)$:

$$\Delta = -d\delta - \delta d = (d-\delta)^2$$

Dirac eq. : $(d-\delta)\Psi = 0 \quad \longleftrightarrow \quad (\gamma^\mu)_{\alpha\beta}\partial_\mu \psi_{\beta(\gamma)} = 0$

$\Uparrow$

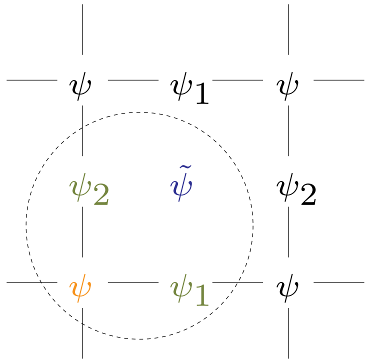
**Dirac-Kähler field** $\Psi$ is expanded by (fermionic) **antisymmetric tensor** fields,

$$\Psi = \psi + \psi_\mu dx^\mu + \frac{1}{2!}\psi_{\mu\nu} dx^\mu \wedge dx^\nu + \cdots$$

$$= \sum_n \frac{1}{n!}\psi_{\alpha(\beta)}(\gamma_{\mu_1\cdots\mu_n})^{\alpha(\beta)} dx^{\mu_1}\wedge\cdots\wedge dx^{\mu_n}$$

$\alpha$: spinor indices, $(\beta)$: **flavor** indices

$\alpha, (\beta) = 1, \cdots, 2^{[D/2]}$




In two dimensions scalar, vector and pseudscalar are associated with the site, link and plaquette, respectively.

$\Downarrow$

The fields of **differential forms** are naturally put on the simplex.

$\Downarrow$

**Generalized Gauge Theory**

# II. Generalized Gauge Theory — Kawamoto-Watabiki

—— Theory formulated by **differential forms**:

**ingredients:** $\begin{pmatrix} \text{even forms} \\ \text{odd forms} \end{pmatrix} \times \begin{pmatrix} \text{bosons} \\ \text{fermions} \end{pmatrix} \Longleftarrow$ **quaternions**

(to control $Z_2$ structures of four variables)

$\mathbf{1},\ \mathbf{i},\ \mathbf{j},\ \mathbf{k} \qquad (\mathbf{ij}=\mathbf{k}, \cdots)$

◇ **gauge field (including fermionic fields):**

$A = T^a \underline{A^a_\mu dx^\mu}$

$$\Longrightarrow \quad \begin{aligned} \mathcal{A} &= \mathbf{1}\psi^\alpha\Sigma^\alpha + \mathbf{i}\hat{\psi}^a T^a + \mathbf{j}A^a T^a + \mathbf{k}\hat{A}^\alpha\Sigma^\alpha \\ &= \mathbf{1}\ (\text{fermionic odd forms}) + \mathbf{i}\ (\text{fermionic even forms}) \\ &\quad + \mathbf{j}\ (\text{bosonic odd forms}) + \mathbf{k}\ (\text{bosonic even forms}) \qquad \in \Lambda_- \end{aligned}$$

$$\begin{cases} A^a = \underline{A^a_\mu dx^\mu} + \frac{1}{3!}A^a_{\mu\nu\rho}dx^\mu dx^\nu dx^\rho + \cdots \\ \hat{A}^\alpha = A^\alpha + \frac{1}{2!}A^\alpha_{\mu\nu}dx^\mu dx^\nu + \cdots \\ \psi^\alpha = \psi^\alpha_\mu dx^\mu + \frac{1}{3!}\psi^\alpha_{\mu\nu\rho}dx^\mu dx^\nu dx^\rho + \cdots \\ \hat{\psi}^a = \psi^a + \frac{1}{2!}\psi^a_{\mu\nu}dx^\mu dx^\nu + \cdots \end{cases}$$

⋆ graded Lie algebra: $[T,T] \sim T,\ \ [T,\Sigma] \sim \Sigma,\ \ \{\Sigma,\Sigma\} \sim T$

even if no fermionic degrees of freedom

◇ **gauge parameter:**

$v = \underline{T^a a^a}$

$$\mathcal{V} = \mathbf{1}\hat{a}^a T^a + \mathbf{i} a^\alpha \Sigma^\alpha + \mathbf{j}\hat{\alpha}^\alpha \Sigma^\alpha + \mathbf{k}\alpha^a T^a$$

$\Longrightarrow$ $= \mathbf{1}$ (bosonic even forms) $+ \mathbf{i}$ (bosonic odd forms)

$+ \mathbf{j}$ (fermionic even forms) $+ \mathbf{k}$ (fermionic odd forms) $\quad \in \Lambda_+$

$\star$ $Z_2$ structure: $\Lambda_+\Lambda_+ \sim \Lambda_+, \quad \Lambda_+\Lambda_- \sim \Lambda_-, \quad \Lambda_-\Lambda_- \sim \Lambda_+,$

◇ **differential operator:**

$d(= dx^\mu \partial_\mu)$

$\Longrightarrow$ $Q = \mathbf{j}d \Big( +\mathbf{k}$(bosonic odd) $+\mathbf{1}$(fermionic odd) $+\mathbf{i}$(fermionic even)$\Big) \quad \in \Lambda_-$

$\longleftarrow$ some space to extension (later)

◇ **gauge transformation:** $\delta\mathcal{A} = [Q + \mathcal{A}, \mathcal{V}]$

## III. Chern-Simons Action — Kawamoto-Watabiki

$$\frac{1}{2}\mathcal{A}Q\mathcal{A} + \frac{1}{3}\mathcal{A}^3 = \mathbf{1}(\text{fermionic odd}) + \mathbf{i}(\text{fermionic even}) + \mathbf{j}(\text{bosonic odd}) + \mathbf{k}(\text{bosonic even}) \quad \in \Lambda_-$$

$$\Longrightarrow \quad S^{\mathrm{e}}_{\mathrm{GCS}} = \int_M \mathrm{Tr}_{\mathbf{k}}\left(\frac{1}{2}\mathcal{A}Q\mathcal{A} + \frac{1}{3}\mathcal{A}^3\right)$$

: on an arbitrary dimensional manifold $M$

◇ relations:

$$\int_M \mathrm{Str}_{\mathbf{1}}\mathcal{F}^2 = \int_M \mathrm{Str}_{\mathbf{1}}\left(Q\left(\mathcal{A}Q\mathcal{A} + \frac{2}{3}\mathcal{A}^3\right)\right)$$

$$\cdots$$

⋆ **Four-dimensional model** (without fermions)

$$S = \int_M \mathrm{Tr}\left\{ \underbrace{B(d\omega + \omega^2)}_{BF\text{-type}} + \phi(d\Omega + \{\Omega, \phi\}) - \phi^2 H - \phi B^2 \right\}$$

gauge field: $\mathcal{A} = \mathbf{j}(\omega + \Omega) + \mathbf{k}(\phi + B + H)$

· gauge transformations:

$$\begin{aligned}
\delta\phi &= [\phi, v], \\
\delta\omega &= dv + [\omega, v] - \{\phi, u\}, \\
\delta B &= du + \{\omega, u\} + [B, v] + [\phi, b], \\
\delta\Omega &= db + [\omega, b] + [\Omega, v] - \{B, u\} + \{\phi, U\}, \\
\delta H &= -dU - \{\omega, U\} + \{\Omega, u\} + [H, v] + [B, b] + [\phi, V],
\end{aligned}$$

with gauge parameters: $\mathcal{V} = \mathbf{1}(v + b + V) + \mathbf{i}(u + U)$

· eq. of motion: $\mathcal{F} = \mathcal{Q}\mathcal{A} + \mathcal{A}^2 = 0$

$$\begin{cases} \phi^2 = 0, \\ d\phi + [\omega, \phi] = 0, \\ d\omega + \omega^2 - [\phi, B] = 0, \\ dB + [\omega, B] + \{\phi, \Omega\} = 0, \\ d\Omega + \{\omega, \Omega\} + [\phi, H] - B^2 = 0. \end{cases}$$

⋆ **classical solutions of eqs. of motion modulo gauge transformation**

— c.f. Horn-Witten (Three-dimensional conformal gravity)

(# of gauge fields) = (# of gauge parameters)
$\Longrightarrow$ topological (conformal gravity)

· gauge group: $sc^+(5,1) \supset SO(5,1)$ (four-dimensional conformal group)

· "order parameter" $\alpha, \beta$ ($\longleftarrow$ solutions for 0-form fields)

(i) $\alpha, \beta \neq 0$: non-gravity phase (all fields $= 0$)

(ii) $\alpha = \beta = 0$: (topological) gravity phase

(gauge trans.) = (diff.) $\oplus$ (local Lorentz) $\oplus$ (conformal trans.)

## ◇ Quantization of Chern-Simons Action — with Kawamoto, Suehiro, Umetsu

$$S_0 = \int \mathrm{Tr}_{\mathbf{k}} \left( \frac{1}{2} \mathcal{A} Q \mathcal{A} + \frac{1}{3} \mathcal{A}^3 \right)$$

eq. of motion: $\mathcal{F} = Q\mathcal{A} + \mathcal{A}^2 = 0$

### ◇ on-shell reducible gauge symmetry

$$\delta\mathcal{A} = [Q + \mathcal{A}, \mathcal{V}] \longrightarrow [Q + \mathcal{A}, \{Q + \mathcal{A}, \mathcal{V}'\}] = [\mathcal{F}, \mathcal{V}'] = 0$$

$$\mathcal{V} = \{Q + \mathcal{A}, \mathcal{V}'\} \longrightarrow \{Q + \mathcal{A}, [Q + \mathcal{A}, \mathcal{V}'']\} = [\mathcal{F}, \mathcal{V}''] = 0$$

$$\mathcal{V}' = [Q + \mathcal{A}, \mathcal{V}''] \longrightarrow \quad \cdots$$

$$\cdots$$

infinite stage

infinite reducible system $\Longrightarrow$ infinite series of ghosts

$\Longleftarrow$

- 0-form field ( cf. two form gauge field $: \delta B = du \longrightarrow \delta u = dv \not\longrightarrow$ )
- algebraic structure of Chern-Simons form

  c.f. (Chern-Simons type) string field theory

## —— Batalin-Vilkovisky formulation (introducing antifields)

The first step is to find the solution $\tilde{S}$ of the master equation,

$$(\tilde{S},\tilde{S}) \equiv \left(\frac{\delta_R \tilde{S}}{\delta\phi^a}\frac{\delta_L \tilde{S}}{\delta\phi^*_a} - \frac{\delta_R \tilde{S}}{\delta\phi^*_a}\frac{\delta_L \tilde{S}}{\delta\phi^a}\right) = 0,$$

under the boundary conditions

$$\begin{cases} \tilde{S}|_{\phi^*=0} = S_0 \quad \text{(classical action)} \\ \frac{\delta\tilde{S}}{\delta\phi^*_{a_n}}|_{\phi^*=0} = Z^{a_n}_{a_{n+1}}\phi^{a_{n+1}} \quad \text{(reducibility conditions)} \end{cases}$$

$\phi^*$: antifields ( (ghost#) $< 0$)

$$\Longrightarrow \quad \tilde{S} = S_0 + \phi^*_{a_0} R^{a_0}_{a_1} C^{a_1} + \phi^*_{a_1}\phi^*_{b_1} E^{b_1 a_1}_{a_2 b_2} C^{b_2} C^{a_2} + \cdots$$

⋆ BRST transformation: $s\phi^a = (\phi^a, \tilde{S})$

⋆ Gauge-fixed action:

$$S_{\text{gauge-fixed}} = \left(\tilde{S} + S_{\text{non-minimal}}\right)\Big|_{\phi^* = \frac{\delta\Psi}{\delta\phi}}$$

$$\begin{cases} S_{\text{non-minimal}} : \text{BRST-trivial sector} \\ \Psi : \text{gauge fermion (including gauge fixing conditions)} \end{cases}$$

**solution:** (minimal sector)

$$\tilde{S} = \int \mathrm{Tr}^{0}_{\mathbf{k}} \left( \frac{1}{2} \tilde{\mathcal{A}} Q \tilde{\mathcal{A}} + \frac{1}{3} \tilde{\mathcal{A}}^3 \right)$$ **same** Chern-Simons form

where $\tilde{\mathcal{A}} = \mathbf{1}\psi + \mathbf{i}\hat{\psi} + \mathbf{j}A + \mathbf{k}\hat{A}$

$$\begin{cases} A = \cdots + C^*_{-2} + \omega + C_2 + \cdots \\ \qquad \nwarrow \quad \nearrow \\ \qquad \text{antifield-field} \\ \qquad \swarrow \quad \searrow \\ \psi = \cdots + C^*_{-3} + \omega^*_{-1} + C_1 + \cdots \end{cases}$$

($\omega$: classical field, $C_1$: ghost, $C_2$: ghost for ghost, $\cdots$)

The generalized gauge field can accommodate an infinite number of fields in a compact form and treat them in **a unified way**.

⋆ **BRST transformation:** extension of differential operator

$$s\tilde{\mathcal{A}} = -\tilde{\mathcal{F}}\mathbf{i}\lambda \quad \longrightarrow \quad (\mathbf{j}d + \mathbf{i}s)\tilde{\mathcal{A}} + \tilde{\mathcal{A}}^2 = 0$$ : flat connection $\qquad Q \Longrightarrow \mathcal{Q} = \mathbf{j}d + \mathbf{i}s$

· nilpotency (⟵ Bianchi identity)

- dimension-independent formulation
- the same result from Hamiltonian formulation

  $\Big(\longrightarrow$ BRST charge is given by fermionic generalized Chern-Simons form $\Big)$
- suitable choice of gauge fixing condition

- loop contribution:
  - · partition function $Z \sim$ constant
  - · UV-free

  $\Big(\longleftarrow$ $\zeta$-function regularization$\Big)$

# IV. Noncommutative Geometry (á la Connes-Lott)

with Kawamoto, Umetsu

**Noncommutative Geometry on $M \times Z_2$**

**(discrete two sheeted manifold)**

**Generalized Gauge Theory**

$$D = \begin{pmatrix} \gamma_\mu \partial_\mu & m\gamma_5 \\ m\gamma_5 & \gamma_\mu \partial_\mu \end{pmatrix} \quad \Longleftrightarrow \quad \mathcal{Q} = \mathbf{j}d + \mathbf{k}m$$

$$\pi(\rho) = \sum_i a_i[D, b_i] = \begin{pmatrix} \gamma_\mu A_\mu & \gamma_5 \phi_{12} \\ \gamma_5 \phi_{21} & \gamma_\mu B_\mu \end{pmatrix} \quad \Longleftrightarrow \quad \mathcal{A} = \mathbf{j}A^a T_a + \mathbf{k}\phi^\alpha \Sigma_\alpha$$

$Z_2$-grading structure on noncommutative geometry is realized by the quaternion structure.

We can choose graded Lie algebra $SU(2|1) \supset SU(2) \times U(1)$ as the gauge algebra $T_a, \Sigma_\alpha$ (without fermionic fields).

$\Longrightarrow$ ( spontaneously broken ) Weinberg-Salam model (with $\theta_W = \pi/6$)
$m$ is V.E.V. of Higgs field

$\Uparrow$

extension of differential op. to **other direction**

# V. Topological Yang-Mills and Dirac-Kähler Fermion

with Kawamoto

Topological Field Theory $\Longleftrightarrow$ Supersymmetric Theory

Topological Twist    Witten

$\Downarrow$

**Generalized Gauge Theory** ($\ni$ fermionic tensor)

$\updownarrow$

**Dirac-Kähler (ghost) fermion**

$\Uparrow$

**Topological Twist**

rearrangement of quantum number

$\Downarrow$

$\mathcal{N} = 2$ **Supersymmetric Theory** ($\ni$ physical fermion)

## ◇Topological Field Theory

— Witten, Labastida-Pernici, Baulieu-Singer

$$S = S_{\text{top}} + \underbrace{s(\text{instanton relation})}_{\text{BRST-exact}}$$

$\uparrow$

$$\int \mathrm{Str}_{\mathbf{1}} \mathcal{F}_0^2 = \int \mathrm{Str}_{\mathbf{1}} Q\Big(\mathcal{A}_0 Q \mathcal{A}_0 + \frac{2}{3}\mathcal{A}_0^3\Big)$$

⋆ **Gauge transformation:** $\delta\mathcal{A}_0 = [Q + \mathcal{A}_0, \mathcal{V}_0] + \underbrace{\mathcal{E}_0}_{\text{shift symmetry}}$ , $\Big(\delta\mathcal{F}_0 = [\mathcal{F}_0, \mathcal{V}_0] + \{Q + \mathcal{A}_0, \mathcal{E}_0\}\Big)$

⋆ **Reducibility:** $\mathcal{V}_0 = \mathcal{V}_1, \quad \mathcal{E}_0 = -[Q + \mathcal{A}_0, \mathcal{V}_1]$

⋆ **Classification of fields** to construct nilpotent BRST algebra (two dimensions)

**classical field**

$\phi, \omega, B$

$$\Longrightarrow \quad \mathcal{A} = \mathbf{j}\omega + \mathbf{k}(\phi + B) + \mathbf{1}C_{(1)} + \mathbf{i}(C_{(0)} + C_{(2)}) \quad \in \Lambda_-$$

**ghost**

$C_{(0)}, C_{(1)}, C_{(2)}$

(FP ghost)

$\tilde{C}_{(0)}, \tilde{C}_{(1)}, \tilde{C}_{(2)}$

(topological ghost)

$$\Longrightarrow \quad \mathcal{C} = \mathbf{j}(\tilde{C}_{(0)} + \tilde{C}_{(2)}) + \mathbf{k}\tilde{C}_{(1)} + \mathbf{1}(\eta_{(0)} + \eta_{(2)}) + \mathbf{i}\eta_{(1)} \quad \in \Lambda_+$$

suffix $(n)$ : the form-degree

**ghost for ghost**

$\eta_{(0)}, \eta_{(1)}, \eta_{(2)}$

⋆ **Extension of differential operator**

$$Q \Longrightarrow \mathcal{Q} = \mathbf{j}d + \mathbf{i}s \qquad \in \Lambda_-$$

↑

BRST op. (fermionic 0-form)
differential to the "ghost" direction

## ◇ BRST algebra

— Witten, Baulieu-Singer, Kanno

⋆ horizontal condition: $\mathcal{F} = \mathcal{Q}\mathcal{A} + \mathcal{A}^2 = \mathcal{F}_0 + \mathcal{C}$

⋆ Bianchi identity: $\mathcal{Q}\mathcal{F} + [\mathcal{A}, \mathcal{F}] = 0$

## ◇ Two-dimensional model ( in Euclidean space )

⋆ gauge algebra (two-dimensional Clifford algebra):

$$\begin{cases} T^a : 1, \gamma_5 \\ \Sigma^\alpha : \gamma^a \quad (a = 1, 2) \end{cases} \qquad \mathrm{Str}(\cdots) = \mathrm{Tr}(\gamma_5 \cdots)$$

⋆ gauge field:

$$\mathcal{A}_0 = \mathbf{j}\gamma_5 \omega_\mu dx^\mu + \mathbf{k}\gamma^a \phi^a$$

⋆ **Instanton relation:**

$$\pm \int \mathrm{Str}\mathcal{F}_0^2$$

$$= \int d^2x \left\{ \frac{1}{2}\left| F_{\mu\nu} \pm \epsilon_{\mu\nu}|\phi|^2 \right|^2 + \frac{1}{2}\left|(D_\mu\phi)_a \pm \epsilon_{\mu\nu}\epsilon_{ab}(D_\nu\phi)_b\right|^2 \right\}$$

$$- \int d^2x \left\{ \frac{1}{2} F_{\mu\nu}F_{\mu\nu} + (D_\mu\phi)_a(D_\mu\phi)_a + |\phi|^4 \right\},$$

where $(D_\mu\phi)_a = \partial_\mu\phi_a - 2\epsilon_{ab}\omega_\mu\phi_b$.

- ⋆ Liouville vortex solution
- ⋆ (dimensionally reduced) Seiberg-Witten relation

⋆ **(partially gauge-fixed) Action:**

$$S = S_{\text{top.}} + s \int d^2x \Big\{ \lambda_{\mu\nu}(F_{\mu\nu} - \epsilon_{\mu\nu}|\phi|^2 + \pi_{\mu\nu})$$

$$+ \chi_{\mu a}\Big((D_\mu\phi)_a + \pi_{\mu a}\Big)$$

$$+ \bar{\eta}\ \partial_\mu \tilde{C}_\mu \Big\}$$

where $\chi_{\mu a} = -\epsilon_{\mu\nu}\epsilon_{ab}\chi_{\nu b}$ (self-dual)

## ◇Twisted algebra

The gauge-fixed action has the following symmetry:

**TFT**

$$\begin{cases} Q_B^2 = 0, \\ + \\ \{Q_B, Q_\mu\} = 2P_\mu, \quad \{\tilde{Q}, Q_\mu\} = -2\epsilon_{\mu\nu}P_\nu, \quad \{Q_\mu, Q_\nu\} = \{\tilde{Q}, \tilde{Q}\} = \{Q_B, \tilde{Q}\} = 0, \\ [J', Q_B] = [J', \tilde{Q}] = 0, \\ [J', Q_\mu] = i\epsilon_{\mu\nu}Q_\nu, \end{cases}$$

$\Uparrow$

**Twist**

$$\begin{cases} J' = J + R \\ Q_{\alpha,i} \rightarrow Q_{\alpha\beta} = \frac{1}{2}\left(\frac{1}{\sqrt{2}}\delta_{\alpha\beta}Q_B + \sqrt{2}(\gamma_\mu)_{\alpha\beta}Q_\mu + \frac{1}{\sqrt{2}}(\gamma_5)_{\alpha\beta}\tilde{Q}\right) \quad \text{(:diagonal sum)} \end{cases}$$

$\Downarrow$

$\mathcal{N} = 2$ **SUSY**

$$\begin{cases} \{Q_{\alpha,i}, Q_{\beta,j}\} = \delta_{ij}(\gamma_\mu)_{\alpha\beta}P_\mu \\ [J, Q_{\alpha,i}] = \frac{i}{2}(\gamma_5)_{\alpha\beta}Q_{\beta,i} \\ [R, Q_{\alpha,i}] = \frac{i}{2}(\gamma_5)_{ij}Q_{\alpha,j} \qquad (\longleftarrow \mathcal{N} = 2\ R\text{-symmetry}) \end{cases}$$

$\alpha(=1,2)$: spinor index, $i(=1,2)$: $\mathcal{N}=2$ index

◇ **Dirac-Kähler ghost**

$$S = \cdots + \underbrace{i\tilde{C}_\mu \partial \rho - i\lambda_{\mu\nu}\partial_\mu \tilde{C}_\nu - i\chi_{\mu a}\partial_\mu \tilde{C}_a}_{} + \cdots$$

$$\updownarrow$$

$$\mathrm{Tr}\Big(i\psi^\dagger \gamma_\mu \partial_\mu \psi + i\chi^\dagger \gamma_\mu \partial_\mu \chi\Big)$$

where **Dirac-Kähler field**:

$$\psi_{\alpha\beta} = \tfrac{1}{2}\Big(\rho + \tilde{C}_\mu \gamma_\mu - \tfrac{1}{2}\lambda_{\mu\nu}\epsilon_{\mu\nu}\gamma_5\Big)_{\alpha\beta \leftarrow \text{flavor}}$$

$$\chi_{\alpha\beta} = \tfrac{1}{2}\Big(-\tilde{C}_{a=1} + \chi_{\mu,a=1}\gamma_\mu - \tilde{C}_{a=2}\gamma_5\Big)_{\alpha\beta}$$

⋆ **$R$-symmetry ($\sim$ flavor symmetry) and Lorentz symmetry:**

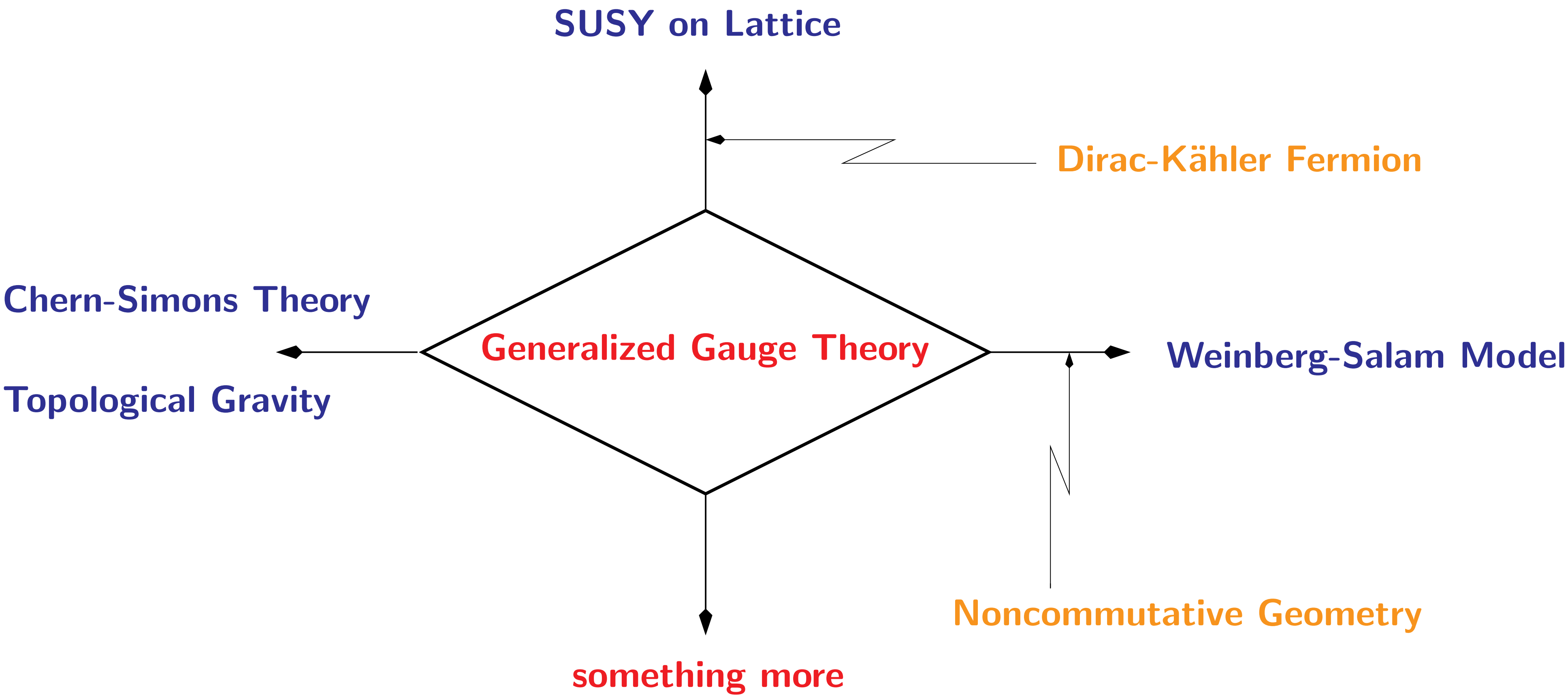
$$\begin{cases} \delta_R \psi = \psi(\tfrac{1}{2}\gamma_5) & \text{:flavor symmetry} \\ \delta_{J'}\psi = -\tfrac{1}{2}[\gamma_5, \psi] & \text{:Lorentz trans.} \end{cases}$$

$$\Longrightarrow \quad \delta_J \psi = \delta_{J'}\psi - \delta_R \psi$$

$$= -\tfrac{1}{2}\gamma_5 \psi \quad \text{:transformation of spinor}$$

**The twisting mechanism can be understood from the Dirac-Kähler formulation** $\Longrightarrow$ **SUSY on lattice**

# VI. Summary and Discussions




◇ Generalized Chern-Simons Theory (more)

$\longrightarrow S_{\mathrm{GCS}} = s(\cdots)$ ($s$-exact, $s^2 = 0$)

**new** fermionic symmetry $s$ exists (with Kawamoto)