Generation of interannual and interdecadal climate oscillations through nonlinear subharmonic resonance in delayed oscillators

Shoshiro Minobe
Division of Earth and Planetary Sciences, Graduate School of Science, Hokkaido University, Sapporo, Japan

Frontier Research System for Global Change, Yokohama, Japan

Fei-fei Jin
Department of Meteorology, Florida State University, Tallahassee, Florida, USA

Received 20 February 2004; revised 30 April 2004; accepted 27 July 2004; published 21 August 2004.

[1] The resonant behavior of delayed oscillators is studied using two simple prototype equations similar to that used by Suarez and Schopf. One prototype equation has a periodic modulation of simultaneous feedback, and the other prototype equation has a periodic external forcing term. The periodic modulation yields even-multiple resonance to the modulation periods, while the periodic forcing results in odd-multiple resonance to the forcing periods. The reason why these two-types of resonance occur in each system is explained. The key mechanism for the resonance is that a positive simultaneous feedback for small amplitudes sets a threshold. Only when the sum of non-simultaneous terms is larger than the threshold, a phase reversal can take place. The implications of El Niño elimination due to annual cycle and potential importance for the decadal variability are discussed. *INDEX TERMS:* 1620 Global Change: Climate dynamics (3309); 3220 Mathematical Geophysics: Nonlinear dynamics; 3339 Meteorology and Atmospheric Dynamics: Ocean/atmosphere interactions (0312, 4504); 4215 Oceanography: General: Climate and interannual variability (3309); 4522 Oceanography: Physical: El Nino. **Citation:** Minobe, S., and F. Jin (2004), Generation of interannual and interdecadal climate oscillations through nonlinear subharmonic resonance in delayed oscillators, *Geophys. Res. Lett.*, *31*, L16206, doi:10.1029/2004GL019776.

1. Introduction

[2] Delay-negative feedback models were widely used for interannual variability, such as El Niño/Southern Oscillation (ENSO) [*Suarez and Schopf*, 1988; *Battisti and Hirst*, 1989] and also for interdecadal variations [*Latif and Barnett*, 1994; *Gu and Philander*, 1997; *Jin*, 1997]. It is known that nonlinear delayed oscillators exhibit interesting subharmonic resonance, which was intensively studied to explain the phase-locking of ENSO to the annual cycle [e.g., *Tziperman et al.*, 1994, 1998; *Jin et al.*, 1994, 1996; *Neelin et al.*, 2000; *Saunders and Ghil*, 2001; *Liu*, 2002]. Also, a subharmonic resonance might be responsible for the postulated resonance feature between the bi-decadal oscillation (about 20-year period) and the penta-decadal oscillation (50–70-year period) for the Pacific sector as suggested from the observations by *Minobe* [1999, 2000], who argued that the simultaneous phase reversals of these two oscillations marked the climatic regime shifts over the North Pacific in the 1920s, 1940s and 1970s, which were reported by *Minobe* [1997] and *Mantua et al.* [1997]. Recently, *Solomon et al.* [2003] reported that a decadal oscillation is resonant with the interannual variability in their hybrid air-sea coupled model.

[3] However, it was not explained adequately how subharmonic resonance occurs and what processes determine the parameter ranges of subharmonic resonance. In the present paper, therefore, we aim to address these questions using simple prototype equations for delayed oscillators.

2. Two Prototype Models

[4] Subharmonic resonance for delayed oscillators occurs, if feedback strength is modulated periodically [*Tziperman et al.*, 1998; *Jin et al.*, 1994, 1996; *Liu*, 2002], or if periodic external forcing is applied [*Tziperman et al.*, 1994; *Wu et al.*, 1993; *Saunders and Ghil*, 2001; *Liu*, 2002]. For clarity, the former case is called *periodic modulation* and the latter case is called *periodic forcing* in the present paper. We use two of the simplest nondimensional prototype equations, which are similar to the delayed oscillator equation proposed by *Suarez and Schopf* [1988], with minimal changes in order to include the periodic modulation and periodic forcing.

[5] The prototype equation for the periodic modulation is as follows

$$\frac{df}{dt} = \left[1 + A_M \cos\left(\frac{2\pi t}{T_M}\right)\right]\left(f - f^3\right) - \alpha f(t - \tau), \qquad (1)$$

where f is the representative amplitude for a physical system, i.e., sea surface-temperature anomaly over the eastern equatorial Pacific for the ENSO, t is the time, $f - f^3$ represents the simultaneous feedback, which consists of linear positive feedback and nonlinear negative feedback, and $\alpha f(t - \tau)$ represents the delay-negative feedback term with the delay coefficient, α, and the delay time, τ. The periodic modulation of the simultaneous feedback is expressed by the terms in the brackets, where A_M is the modulation amplitude, and T_M is the modulation period.

[6] The other prototype equation, which is for the periodic forcing, is given by

$$\frac{df}{dt} = f - f^3 - \alpha f(t - \tau) + A_F \cos\left(\frac{2\pi t}{T_F}\right), \qquad (2)$$

0094-8276/04/2004GL019776$05.00

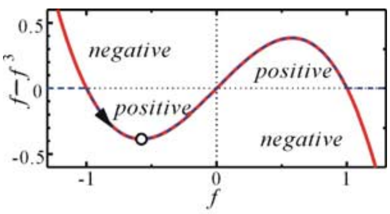

Figure 1. Simultaneous feedback as a function of amplitude. Simultaneous feedback of $f - f^3$ is shown red curve, and the same feedback but neutral for $|f| > 1$ is shown in blue dashed curve. When the amplitude changes from -1 to $+1$, the simultaneous feedback takes its negative maximum of -0.38 (open circle), whose absolute value is the threshold. This threshold feature is unchanged for the feedback that is neutral for large amplitudes. Note that the first and third (second and fourth) quadrants are associated with the positive (negative) feedback.

where the last term is the forcing term, A_F is the forcing amplitude, and T_F is the forcing period. *Wu et al.* [1993] suggested that the role of annual period winds for ENSOs is expressed as the external forcing term.

3. Threshold Arising From Positive Feedback

[7] A key feature of equations (1) or (2) is the positive simultaneous feedback for small amplitudes. The positive feedback for small amplitudes must be accompanied by negative or neutral feedback for large amplitudes, so that the system does not goes to a runaway situation. In order to explain a role of these feedbacks, we introduce the following equation;

$$\frac{df}{dt} = f - f^3 + G, \tag{3}$$

where G is the sum of non-simultaneous terms, which can consist of delay term and forcing term.

[8] Consider a situation where amplitude, f, changes its sign from a negative value around -1 to a positive value around $+1$ (Figure 1). Along this course, df/dt should be positive, and hence the right hand side must also be positive, i.e.,

$$G > -\max\left(f - f^3\right) = 2/3^{3/2} \approx 0.38 \tag{4}$$

Equation (4) indicates that the sum of the non-simultaneous terms must be larger than a *threshold* value (0.38) for a phase reversal, during a period long enough that the system can respond (longer than nondimensional unit time). It should be noted that this threshold feature does not qualitatively depend on details of the formulation of the simultaneous feedback terms, as long as the simultaneous feedback is positive for small amplitudes. For example, if the feedback for large amplitudes is neutral (blue dashed line in Figure 1), the threshold nature is unchanged. As explained below, the threshold plays a central role in the subharmonic resonance for both the periodic modulation and periodic forcing.

4. Resonance to Periodic Modulation

[9] We have integrated equation (1) for nondimensional time of 1000 from initial amplitude of unity. Integration is conducted for each delay time, which varies from 0.1 to 12.5 with a step of 0.1, with the delay coefficient, α, of 0.5, 0.75, and 1. The modulation parameters are set as $A_M = 1$, $T_M = 4$.The unit timescale for nondimensionalization is the dimensional linear simultaneous feedback time [*Suarez and Schopf*, 1988]. Therefore, in the case of the annual modulation of ENSOs, $T_M = 4$ means that the simultaneous feedback has the dimensional e-folding time of three months (1/4 yr). If we assume the physical delay time is about a half year [*Battisti and Hirst*, 1989], then the nondimensional delay time becomes two. We explore a wider range of delay times, in order to reach better understanding of resonance structure.

[10] Figure 2 summarizes the relation between the delay times and mean periods, which are calculated from the zero-crossing points for the latter half of the numerical integration. The overall structure exhibits a feature similar to *Devil's staircase* characterized by wide and narrow steps at certain periods with a monotonic increase of periods as delay increases. Consistent with the previous studies [e.g., *Jin et al.*, 1994; *Neelin et al.*, 2000; *Liu*, 2002], the steps are wide for even-multiples of the modulation period. The step widths are wider for smaller delay coefficients, and are approximately the same between quadruple- and sextuple-period resonance for a constant delay coefficient.

[11] The reason why the even-period resonance is dominant for the periodic modulation is explained by taking account of the threshold. The periodic modulation means that the threshold is weakened at every one period of the modulation. With a moderate delay coefficient, α, the amplitude can reverse its phase, only when the threshold is weak. If the phase reverses with each (every two) of the weakened thresholds, the resultant period becomes the double (quadruple) of the modulation period. Consequently, the periodic modulation results in dominant even-multiple resonance. When delay coefficient and hence delay term become large, the chance that the delay term overwhelms the threshold increases, and the oscillations are less constrained by the modulation. Therefore, as the delay coeffi-

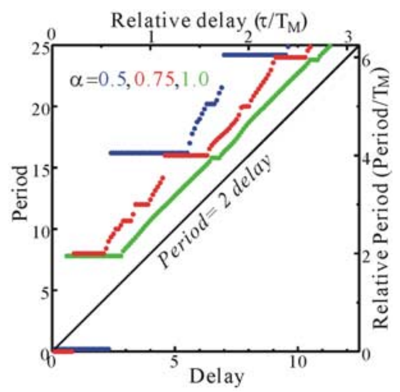

Figure 2. Mean periods as a function of delay times for the periodic modulation ($\mathbf{T_M} = 4$, $\mathbf{A_M} = 1$). The right hand axis indicates the oscillation period relative to the modulation period. The blue, red and green dots are for the delay coefficient, α, of 0.5, 0.75, and 1.0, respectively. The blue (green) dots are shifted +0.2 (-0.2) for the left hand axis for an easier visual inspection.

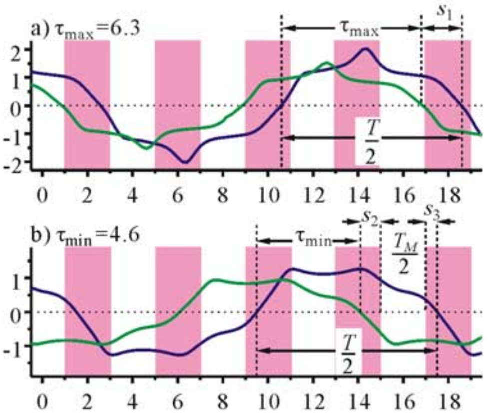

Figure 3. Time series of f (blue curve) and delay term (green curve) along with the modulation strength (pink shades) at the longest (a) and shortest (b) delay times for the quadruple-period resonance for the periodic modulation. The pink shades indicate where the modulation, given by the terms in the bracket in (1), is weaker than unity. The delay coefficient, α, is 0.75, and the other parameters are the same as those for Figure 2. For the explanation of the scales such as τ_{max}, τ_{min}, T_M, s_1, s_2, and s_3, see text.

cient is decreased (increased), step widths become wider (narrower) as mentioned above.

[12] It is interesting to examine temporal relation among the terms of the prototype equation (1) for the resonance. Figure 3 shows the time series of the oscillation amplitudes, delay term, and modulation strength at the longest and shortest delay times for the quadruple-period resonance. At the longest delay time (τ_{max}), a half period (T/2) is given by the sum of the delay time and a transient time (s_1) (Figure 3a), i.e., $\tau_{max} + s_1 = T/2$. The transient time s_1 is defined as a time for which the amplitude becomes zero after the phase reversal of the delay term under the condition of the weakened threshold.

[13] Similarly, a half period for the shortest delay time (τ_{min}) is divided as $\tau_{min} + T_M/2 + s_2 + s_3 = T/2$ (Figure 3b), where s_2 is the time during which the oscillation amplitude stays the opposing polarity against the delay term under a weakened threshold. In this case, the phase reversal of the delay term does not directly lead the phase reversal of the oscillation amplitude; before the oscillation amplitude crosses zero, the threshold is strengthened and prohibits a phase reversal for a half of the modulation period. When the threshold is weakened again, the oscillation can reverse its phase after another transient time, s_3. For the n'th subharmonic, the oscillation period is identical to the n'th multiple of the modulation period ($T = nT_M$). Thus, the longest and shortest delay times for the resonance can be written as

$$\tau_{max} = T_M n/2 - s_1 \quad \text{and} \quad \tau_{min} = T_M(n-1)/2 - s_2 - s_3, \quad (5)$$

where n is an even number. Consequently, the step width is given by

$$T_M/2 - s_1 + s_2 + s_3. \quad (6)$$

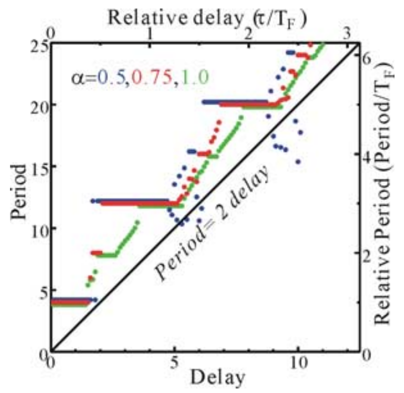

Figure 4. Same as Figure 2, but for the periodic forcing case ($\mathbf{T_F}$ = 4, $\mathbf{A_F}$ = 1).

The transition times, s_1, s_2, and s_3 substantially depend not on the delay times but on the delay coefficients, resulting in the aforementioned roughly constant step width at even-multiple resonance for respective delay coefficients.

5. Resonance to Periodic Forcing

[14] Figure 4 shows mean periods for the numerical integration of the prototype equation for the periodic forcing equation (2), with the forcing parameters of A_F = 1 and T_F = 4, and the other parameters are the same as those for Figure 2. Again, a similar feature to the Devil's staircase is generally evident, but wider steps of oscillation periods are now found at odd-multiples of the forcing period instead of the even-multiples of the modulation period consistent with previous studies [*Wu et al.*, 1993; *Saunders and Ghil*, 2001; *Liu*, 2002].

[15] Figure 5 shows time series for the triple-period resonance at the longest and shortest delay times for the delay coefficient of 0.75. The delay and forcing terms are of similar magnitude (~1.0). Thus, the phase reversal can occur when the delay and forcing terms are in-phase. Two successive phase reversals must have opposing polarity, and the forcing term has the opposing polarity at 0.5, 1.5 and

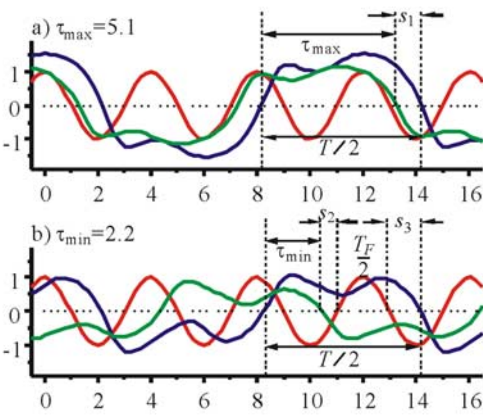

Figure 5. Same as Figure 3, but for the periodic forcing case. The time series of the forcing term is denoted by the red curve.

2.5 forcing periods. This constrains that the oscillation takes 1-, 3-, or 5- subharmonic of the forcing period. Consequently, the periodic forcing results in the odd-multiple resonance.

[16] As in the case of the periodic resonance, the temporal relation of phase reversal with the delay and forcing terms is examined for the longest and shortest delays for the triple-period resonance (Figure 5). In general, the half period is subdivided in a similar manner to the case of the periodic modulation (compare Figures 3 and 5). Therefore, essentially the same equation as that of the periodic modulation gives the longest and shortest delay times for the periodic forcing as,

$$\tau_{\max} = T_F n/2 - s_1 \quad \text{and} \quad \tau_{\min} = T_F(n-1)/2 - s_2 - s_3, \quad (7)$$

where n is an odd number instead of the even number for the periodic modulation.

[17] A stronger delay coefficient may result in a free oscillation, whereas a stronger forcing may yield the oscillation period identical to the forcing period. Consistently, using a slightly more complex model than the present one, *Liu* [2002] reported that a strong annual-period forcing eliminates interannual ENSO variability. He suggested that this mechanism may contributed less active ENSOs in mid-Holocene. The present results indicate that the ENSO elimination may occur, if the annual forcing is larger than the delay term by the threshold value.

6. Discussion

[18] The present study showed that a key mechanism for the resonance in the delayed oscillators is the threshold, which arises from the combination of the simultaneous positive feedback for small amplitudes and negative-or-neutral feedback for large amplitudes. We employed the minimal changes from the delayed oscillator model by *Suarez and Schopf* [1988], and examined a limited parameter space. Larger changes in formulations and wider parameter ranges are worth to be closely explored in future, but some features of different formulations are described here. For example, not only the simultaneous feedback term, but also the delay term may be modulated periodically; in this case, we found that even-number resonance is also dominant. The present simultaneous feedback term, $f - f^3$, is a simplification of a hyperbolic tangent form for an ENSO modeling [*Battisti and Hirst*, 1989]. If we used the hyperbolic tangent function for the simultaneous feedback and delay terms, we still obtained even number resonance for the periodic modulation and odd-number resonance for the periodic forcing.

[19] For ENSO, the periodic modulation and periodic forcing are considered being associated with the annual cycle [e.g., *Liu*, 2002]. In this case, the nondimensional modulation and forcing periods are around 2–3 according to the present nondimensionalization by *Suarez and Schopf* [1988] with parameters of *Battisti and Hirst* [1989]. This is slightly smaller than 4 used for the present integrations. For the periodic modulation, when we use the smaller modulation period of 2, the quadruple- and sextuple-period resonance also occurred, but the double-period resonance, i.e., the highest frequency subharmonic resonance, did not occur. Similarly, for the periodic forcing, triple-period resonance disappeared with the forcing period of 2, but occurred with the forcing period of 3.

[20] It is noteworthy that the dimensional amplitude of the positive linear feedback required for a resonance becomes small as the timescale of phenomena becomes long. This is because the nondimensional period of the modulation (forcing) is equivalent to the ratio of the dimensional period of the modulation (forcing) relative to the e-folding timescale of the linear feedback. Therefore, a weak positive feedback, which is not important in the interannual variations, can result in resonance on interdecadal timescales. For example, the nondimensional forcing period of four, used in the present paper, means that the e-folding timescale of the dimensional linear feedback is quarter of the dimensional forcing period. Thus, when we examine the possibility that an interdecadal oscillation (i.e., 20-yr period) acts as a forcing, the dimensional e-folding time of the positive feedback as small as 5-yrs is large enough to cause a resonance, and has a possibility to yield a 60-yr oscillation. Recently, *Schneider et al.* [2002] reported that a week positive feedback is at work on the timescale of several years in decadal interaction between the Kuroshio/Oyashio extension and Aleutian low analyzing a coupled general circulation model. Their result suggests the possibility of weak positive feedback in the real world. The present study indicates that the positive feedback in the climate system, even if it is small, has much implication.

[21] **Acknowledgments.** We thank anonymous reviewers for their constructive comments, D. S. Battisti for fruitful discussion. This study was supported by grant-in-aid for scientific research (kaken-hi #15540417, to SM) and by 21st century center of excellence program on "Neo-Science of Natural History" lead by H. Okada (to SM) both from the Ministry of Education, Culture, Sports, Science and Technology, Japan.

References

Battisti, D. S., and A. C. Hirst (1989), Interannual variability in a tropical atmosphere-ocean model: Influence of the basic state, ocean geometry, and nonlinearity, *J. Atmos. Sci.*, *46*, 1687–1712.

Gu, D. F., and S. G. H. Philander (1997), Interdecadal climate fluctuations that depend on exchanges between the tropics and extratropics, *Science*, *275*, 805–807.

Jin, F.-F. (1997), A theory of interdecadal climate variability of the North Pacific Ocean-atmosphere system, *J. Clim.*, *10*, 324–338.

Jin, F.-F., J. D. Neelin, and M. Ghil (1994), El Niño on the devil's staircase: Annual subharmonic steps to chaos, *Science*, *264*, 70–72.

Jin, F.-F., J. D. Neelin, and M. Ghil (1996), El Niño/Southern Oscillation and the annual cycle: Subharmonic frequency-locking and aperiodicity, *Physica D*, *98*, 442–465.

Latif, M., and T. P. Barnett (1994), Causes of decadal climate variability over the North Pacific and North America, *Science*, *266*, 634–637.

Liu, Z. (2002), A simple model study of ENSO suppression by external periodic forcing, *J. Clim.*, *15*, 1088–1098.

Mantua, N. J., S. R. Hare, Y. Zhang, J. M. Wallace, and R. C. Francis (1997), A Pacific interdecadal climate oscillation with impacts on salmon production, *Bull. Am. Meteorol. Soc.*, *76*, 1069–1079.

Minobe, S. (1997), A 50–70 year climatic oscillation over the North Pacific and North America, *Geophys. Res. Lett.*, *24*, 683–686.

Minobe, S. (1999), Resonance in bidecadal and pentadecadal climate oscillations over the North Pacific: Role in climatic regime shifts, *Geophys. Res. Lett.*, *26*, 855–858.

Minobe, S. (2000), Spatio-temporal structure of the pentadecadal variability over the North Pacific, *Prog. Oceanogr.*, *47*, 381–408.

Neelin, J. D., F.-F. Jin, and H.-H. Suy (2000), Variations in ENSO phase locking, *J. Clim.*, *13*, 2570–2590.

Saunders, A., and M. Ghil (2001), A Boolean delay equation model of ENSO variability, *Physica D*, *160*, 54–78.

Schneider, N., A. J. Miller, and D. W. Pierce (2002), Anatomy of North Pacific decadal variability, *J. Clim.*, *15*, 586–605.

Solomon, A., J. P. McCreary, R. Kleeman, and B. A. Klinger (2003), Interannual decadal variability in an intermediate coupled model of the Pacific region, *J. Clim.*, *16*, 383–405.

Suarez, M. J., and P. S. Schopf (1988), A delayed action oscillator for ENSO, *J. Atmos. Sci.*, *45*, 3283–3287.

Tziperman, E., L. Stone, M. A. Cane, and H. Jarosh (1994), El Niño chaos: Overlapping of resonances between the seasonal cycle and the Pacific Ocean-atmosphere oscillator, *Science*, *264*, 72–74.

Tziperman, E., M. A. Cane, S. E. Zebiak, Y. Xue, and B. Blumenthal (1998), Locking of El Niño's peak time to the end of the calendar year in the delayed oscillator picture of ENSO, *J. Clim.*, *11*, 2191–2199.

Wu, D.-H., D. L. T. Anderson, and M. K. Davey (1993), ENSO variability and external impacts, *J. Clim.*, *6*, 1703–1717.

F. Jin, Department of Meteorology, Florida State University, Tallahassee, FL, USA.

S. Minobe, Division of Earth and Planetary Sciences, Graduate School of Science, Hokkaido University, Sapporo 060-0810, Japan. (minobe@ep.sci.hokudai.ac.jp)